1a $f(x)=\sin(\boxed{x^2+x})\Rightarrow f'(x)=\cos(x^2+x)\cdot(2x+1)=(2x+1)\cdot\cos(x^2+x)$.

1b $G(x)=f(x)+3\Rightarrow G'(x)=f'(x)=(2x+1)\cdot\cos(x^2+x)=g(x)$.

$f'(x)=\frac{\mathrm{d}f(x)}{\mathrm{d}x}$
dus
$\mathrm{d}f(x)=f'(x)\mathrm{d}x$

$f'(x)\mathrm{d}x=\mathrm{d}f(x)$
dus
$g(x)\mathrm{d}x=\mathrm{d}G(x)$

2a $f(x)=6x\cdot(x^2+1)^5$ is van de vorm $y=f(u)\cdot u'$ met $f(u)=3u^5$ en $u'=2x$, maar $h(x)=6x\cdot(x^3+1)^5$ is dat niet.

2b $g(x)=\frac{10x^3}{\sqrt{x^4+7}}$ is van de vorm $y=g(u)\cdot u'$ met $g(u)=\frac{2\frac{1}{2}}{\sqrt{u}}$ en $u'=4x^3$, maar $k(x)=\frac{10x^3}{\sqrt{x^4+x}}$ is dat niet.

2c Het primitiveren van $l(x)=\frac{10x^3+a}{\sqrt{x^4+x}}$ lukt als $10x^3+a=c\cdot(4x^3+1)\Rightarrow 10=4c$ én $a=c\Rightarrow c=2\frac{1}{2}$ en $a=c\Rightarrow a=2\frac{1}{2}$.

3a $f(x)\mathrm{d}x=\sqrt{\boxed{4x-1}}\,\mathrm{d}x=\frac{1}{4}\cdot\sqrt{\boxed{4x-1}}\,\mathrm{d}(\boxed{4x-1})=\frac{1}{4}\cdot(\boxed{4x-1})^{\frac{1}{2}}\mathrm{d}(\boxed{4x-1})=\mathrm{d}\left(\frac{1}{4}\cdot\frac{2}{3}\cdot(4x-1)^{1\frac{1}{2}}\right)$.

Dus $F(x)=\frac{1}{4}\cdot\frac{2}{3}\cdot(4x-1)^{1\frac{1}{2}}+c=\frac{1}{6}(4x-1)\cdot\sqrt{4x-1}+c$.

3b $\frac{1}{a}\cdot F(ax+b)+c$ zijn de primitieven van $f(ax+b)$ heeft de voorkeur.

$f(x)=\sqrt{4x-1}=(\boxed{4x-1})^{\frac{1}{2}}\Rightarrow F(x)=\frac{1}{4}\cdot\frac{1}{1\frac{1}{2}}\cdot(4x-1)^{1\frac{1}{2}}+c=\frac{1}{6}(4x-1)\cdot\sqrt{4x-1}+c$.

4a $f(x)\mathrm{d}x=2x(\boxed{x^2+4})^3\,\mathrm{d}x=(\boxed{x^2+4})^3\,\mathrm{d}(\boxed{x^2+4})=\mathrm{d}\left(\frac{1}{4}(x^2+4)^4\right)$. Dus $F(x)=\frac{1}{4}(x^2+4)^4+c$.

4b $g(x)\mathrm{d}x=6x\cdot\sqrt{\boxed{x^2+1}}\,\mathrm{d}x=3\cdot 2x\cdot\sqrt{\boxed{x^2+1}}\,\mathrm{d}x=3\cdot\sqrt{\boxed{x^2+1}}\,\mathrm{d}(\boxed{x^2+1})=3\cdot\boxed{x^2+1}^{\frac{1}{2}}\,\mathrm{d}(\boxed{x^2+1})=\mathrm{d}\left(3\cdot\frac{1}{1\frac{1}{2}}(x^2+1)^{1\frac{1}{2}}\right)$.

Dus $G(x)=2(x^2+1)^{1\frac{1}{2}}+c=2(x^2+1)\cdot\sqrt{x^2+1}+c$.

4c $h(x)\mathrm{d}x=6x^2\cdot(\boxed{x^3-1})^4\,\mathrm{d}x=2\cdot(\boxed{x^3-1})^4\,\mathrm{d}(\boxed{x^3-1})=\mathrm{d}\left(2\cdot\frac{1}{5}(x^3-1)^5\right)$. Dus $H(x)=\frac{2}{5}(x^3-1)^5+c$.

4d $j(x)\mathrm{d}x=3x^2\cdot\sin(\boxed{x^3-1})\mathrm{d}x=\sin(\boxed{x^3-1})\mathrm{d}(\boxed{x^3-1})=\mathrm{d}\left(-\cos(x^3-1)\right)$. Dus $J(x)=-\cos(x^3-1)+c$.

5a $f(x)=(\boxed{3x-4})^3\Rightarrow F(x)=\frac{1}{3}\cdot\frac{1}{4}(3x-4)^4+c=\frac{1}{12}(3x-4)^4+c$.

5b $f(x)=(2x-3)\cdot\sqrt{2x-3}=(\boxed{2x-3})^{1\frac{1}{2}}\Rightarrow F(x)=\frac{1}{2}\cdot\frac{1}{2\frac{1}{2}}(2x-3)^{2\frac{1}{2}}+c=\frac{1}{5}(2x-3)^2\cdot\sqrt{2x-3}+c$.

5c $f(x)=\frac{2}{\sqrt{\boxed{1-x}}}=4\cdot\frac{1}{2\sqrt{\boxed{1-x}}}\Rightarrow F(x)=\frac{1}{-1}\cdot 4\cdot\sqrt{1-x}+c=-4\cdot\sqrt{1-x}+c$.

5d $f(x)\mathrm{d}x=\frac{2x}{\boxed{2-3x^2}}\,\mathrm{d}x=-\frac{1}{3}\cdot\frac{1}{\boxed{2-3x^2}}\,\mathrm{d}(\boxed{2-3x^2})=\mathrm{d}\left(-\frac{1}{3}\cdot\ln(|2-3x^2|)\right)$. Dus $F(x)=-\frac{1}{3}\cdot\ln|2-3x^2|+c$.

5e $f(x)=\ln(\boxed{4x+1})\Rightarrow F(x)=\frac{1}{4}\cdot((4x+1)\cdot\ln(4x+1)-(4x+1))+c=\frac{1}{4}\cdot(4x+1)\cdot\ln(4x+1)-\frac{1}{4}\cdot(4x+1)+c$.

5f $f(x)\mathrm{d}x=x\cdot\ln(\boxed{x^2+1})\mathrm{d}x=\frac{1}{2}\cdot\ln(\boxed{x^2+1})\mathrm{d}(\boxed{x^2+1})=\mathrm{d}\left(\frac{1}{2}\cdot\left((x^2+1)\cdot\ln(x^2+1)-(x^2+1)\right)\right)$.

Dus $F(x)=\frac{1}{2}\cdot\left((x^2+1)\cdot\ln(x^2+1)-(x^2+1)\right)+c$.

6a $f(x)\mathrm{d}x=\frac{\boxed{\ln(x)}}{x}\,\mathrm{d}x=\boxed{\ln(x)}\cdot\frac{1}{x}\,\mathrm{d}x=\boxed{\ln(x)}\,\mathrm{d}\left(\boxed{\ln(x)}\right)=\mathrm{d}\left(\frac{1}{2}\cdot(\ln(x))^2\right)$. Dus $F(x)=\frac{1}{2}\cdot\ln^2(x)+c$.

6b $g(x)\mathrm{d}x=x\cdot e^{\boxed{-x^2}}\,\mathrm{d}x=-\frac{1}{2}\cdot e^{\boxed{-x^2}}\,\mathrm{d}\left(\boxed{-x^2}\right)=\mathrm{d}\left(-\frac{1}{2}\cdot e^{-x^2}\right)$. Dus $G(x)=-\frac{1}{2}\cdot e^{-x^2}+c$.

6c $h(x)\mathrm{d}x=x\cdot\sqrt{\boxed{5-x^2}}\,\mathrm{d}x=-\frac{1}{2}\cdot(\boxed{5-x^2})^{\frac{1}{2}}\,\mathrm{d}(\boxed{5-x^2})=\mathrm{d}\left(-\frac{1}{2}\cdot\frac{1}{1\frac{1}{2}}(5-x^2)^{1\frac{1}{2}}\right)$. Dus $H(x)=-\frac{1}{3}\cdot(5-x^2)\cdot\sqrt{5-x^2}+c$.

6d $j(x)\mathrm{d}x=\frac{x}{\sqrt{\boxed{x^2+1}}}\,\mathrm{d}x=\frac{1}{2}\cdot\frac{1}{\sqrt{\boxed{x^2+1}}}\,\mathrm{d}(\boxed{x^2+1})=\frac{1}{2\sqrt{\boxed{x^2+1}}}\,\mathrm{d}(\boxed{x^2+1})=\mathrm{d}\left(\sqrt{x^2+1}\right)$. Dus $J(x)=\sqrt{x^2+1}+c$.

7 $f(x)=\ln(\boxed{\cos(x)})\Rightarrow f'(x)=\frac{1}{\cos(x)}\cdot-\sin(x)=-\frac{\sin(x)}{\cos(x)}=-\tan(x)$.

8a $\tan(\boxed{2x})\,dx = \frac{1}{2}\cdot\tan(\boxed{2x})\,d(\boxed{2x})$ (zie vraag 7) $= \frac{1}{2}\cdot d\left(-\ln|\cos(2x)|\right) = d\left(-\frac{1}{2}\ln|\cos(2x)|\right)$.

$$\int_0^{\frac{1}{6}\pi}\tan(2x)\,dx = \left[-\frac{1}{2}\cdot\ln|\cos(2x)|\right]_0^{\frac{1}{6}\pi} = -\frac{1}{2}\cdot\ln\left|\cos(\tfrac{1}{3}\pi)\right| + \frac{1}{2}\cdot\ln|\cos(0)| = -\frac{1}{2}\cdot\ln(\tfrac{1}{2}) + \ln(1) = -\frac{1}{2}\cdot\ln(2^{-1}) + 0 = \frac{1}{2}\cdot\ln(2).$$

8b $\sin^3(x)\,dx = \sin^2(x)\cdot\sin(x)\,dx$

$$= \left(1-(\boxed{\cos(x)})^2\right)\cdot d\left(-\boxed{\cos(x)}\right) = \left((\boxed{\cos(x)})^2-1\right)\cdot d\left(\boxed{\cos(x)}\right) = d\left(\tfrac{1}{3}(\cos(x))^3-\cos(x)\right).$$

$$\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\sin^3(x)\,dx = \left[\frac{1}{3}\cdot\cos^3(x)-\cos(x)\right]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} = \frac{1}{3}\cdot\cos^3(\tfrac{1}{3}\pi)-\cos(\tfrac{1}{3}\pi)-\left(\frac{1}{3}\cdot\cos^3(\tfrac{1}{4}\pi)-(\cos(\tfrac{1}{4}\pi)\right)$$

$$= \frac{1}{3}\cdot(\tfrac{1}{2})^3-\frac{1}{2}-\left(\frac{1}{3}\cdot(\tfrac{1}{2}\sqrt{2})^3-\frac{1}{2}\sqrt{2}\right) = \frac{1}{3}\cdot\frac{1}{8}-\frac{1}{2}-\frac{1}{3}\cdot\frac{1}{8}\cdot2\sqrt{2}+\frac{1}{2}\sqrt{2} = \frac{1}{24}-\frac{1}{2}-\frac{1}{12}\sqrt{2}+\frac{1}{2}\sqrt{2} = -\frac{11}{24}+\frac{5}{12}\sqrt{2}.$$

8c $\sin^2(x)\cdot\cos(x)\,dx = \left(\boxed{\sin(x)}\right)^2 d(\boxed{\sin(x)}) = d\left(\frac{1}{3}(\sin(x))^3\right)$.

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\sin^2(x)\cdot\cos(x)\,dx = \left[\frac{1}{3}\cdot\sin^3(x)\right]_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} = \frac{1}{3}\cdot\sin^3(\tfrac{1}{2}\pi)-\frac{1}{3}\cdot\sin^3(\tfrac{1}{6}\pi) = \frac{1}{3}\cdot1^3-\frac{1}{3}\cdot(\tfrac{1}{2})^3 = \frac{1}{3}-\frac{1}{3}\cdot\frac{1}{8} = \frac{1}{3}-\frac{1}{24} = \frac{7}{24}.$$

8d $\sin(2x)\cdot\cos(x)\,dx = 2\sin(x)\cdot\cos(x)\cdot\cos(x)\,dx = -2\cdot\left(\boxed{\cos(x)}\right)^2 d\boxed{\cos(x)} = d\left(-\frac{2}{3}\cos(x)^3\right)$.

$$\int_0^{\frac{1}{6}\pi}\sin(2x)\cdot\cos(x)\,dx = \left[-\frac{2}{3}\cos^3(x)\right]_0^{\frac{1}{6}\pi} = -\frac{2}{3}\cos^3(\tfrac{1}{6}\pi)+\frac{2}{3}\cos^3(0) = -\frac{2}{3}\cdot(\tfrac{1}{2}\sqrt{3})^3+\frac{2}{3}\cdot1^3 = -\frac{2}{3}\cdot\frac{1}{8}\cdot3\sqrt{3}+\frac{2}{3} = \frac{2}{3}-\frac{1}{4}\sqrt{3}.$$

8e $$\int_0^1\frac{2}{\sqrt{\boxed{3x+1}}}\,dx = \int_0^1 4\cdot\frac{1}{2\sqrt{\boxed{3x+1}}}\,dx = \left[\frac{1}{3}\cdot4\cdot\sqrt{3x+1}\right]_0^1 = \frac{4}{3}\cdot\sqrt{4}-\frac{4}{3}\cdot\sqrt{1} = \frac{4}{3}\cdot2-\frac{4}{3}\cdot1 = \frac{4}{3}.$$

8f $$\frac{x}{\boxed{x^2+1}}\,dx = \frac{1}{2}\cdot\frac{1}{\boxed{x^2+1}}\,d(\boxed{x^2+1}) = d(\tfrac{1}{2}\cdot\ln(x^2+1)) \Rightarrow \int_0^1\frac{x}{\sqrt{x^2+1}}\,dx = \left[\frac{1}{2}\cdot\ln(x^2+1)\right]_0^1 = \frac{1}{2}\cdot\ln(1^2+1)-\frac{1}{2}\cdot\ln(1) = \frac{1}{2}\cdot\ln(2).$$

8g $$x\cdot\sqrt{\boxed{x^2+1}}\,dx = \frac{1}{2}\cdot\sqrt{\boxed{x^2+1}}\,d(\boxed{x^2+1}) = \frac{1}{2}\cdot(\boxed{x^2+1})^{\frac{1}{2}}\,d(\boxed{x^2+1}) = d\left(\frac{1}{2}\cdot\frac{1}{1\frac{1}{2}}\cdot(x^2+1)^{1\frac{1}{2}}\right) = d\left(\frac{1}{3}\cdot(x^2+1)^{1\frac{1}{2}}\right).$$

$$\int_0^{2\sqrt{2}}x\cdot\sqrt{x^2+1}\,dx = \left[\frac{1}{3}\cdot(x^2+1)\cdot\sqrt{x^2+1}\right]_0^{2\sqrt{2}} = \frac{1}{3}\cdot(8+1)\cdot\sqrt{8+1}-\frac{1}{3}\cdot(0+1)\cdot\sqrt{0+1} = \frac{1}{3}\cdot9\cdot3-\frac{1}{3}\cdot1\cdot1 = 9-\frac{1}{3} = 8\frac{2}{3}.$$

8h $$\frac{e^{\boxed{\sqrt{x}}}}{\sqrt{x}}\,dx = 2e^{\boxed{\sqrt{x}}}\cdot\frac{1}{2\sqrt{x}}\,dx = 2e^{\boxed{\sqrt{x}}}\,d(\boxed{\sqrt{x}}) = d\left(2e^{\sqrt{x}}\right) \Rightarrow \int_1^4\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \left[2e^{\sqrt{x}}\right]_1^4 = 2e^{\sqrt{4}}-2e^{\sqrt{1}} = 2e^2-2e.$$

8i $$x\cdot\ln(\boxed{x^2+1})\,dx = \frac{1}{2}\cdot\ln(\boxed{x^2+1})\,d(\boxed{x^2+1}) = d\left(\frac{1}{2}\cdot\left((x^2+1)\cdot\ln(x^2+1)-(x^2+1)\right)\right).$$

$$\int_1^2 x\cdot\ln(x^2+1)\,dx = \left[\frac{1}{2}\cdot\left((x^2+1)\cdot\ln(x^2+1)-(x^2+1)\right)\right]_1^2 = \frac{1}{2}\cdot(5\cdot\ln(5)-5)-\frac{1}{2}\cdot(2\cdot\ln(2)-2) = 2\frac{1}{2}\cdot\ln(5)-\ln(2)-1\frac{1}{2}.$$

9a $$\frac{\ln^2(x)}{x}\,dx = \left(\boxed{\ln(x)}\right)^2\cdot\frac{1}{x}\,dx = \left(\boxed{\ln(x)}\right)^2 d\left(\boxed{\ln(x)}\right) = d\left(\frac{1}{3}(\ln(x))^3\right).$$

$$\int_1^2\frac{\ln^2(x)}{x}\,dx = \left[\frac{1}{3}\ln^3(x)\right]_1^2 = \frac{1}{3}\ln^3(2)-\frac{1}{3}\ln^3(1) = \frac{1}{3}\ln^3(2).$$

9b $$\frac{1}{x}\cdot\frac{1}{\boxed{\ln(x)}}\,dx = \frac{1}{\boxed{\ln(x)}}\,d\left(\boxed{\ln(x)}\right) = d\left(\ln\left|\boxed{\ln(x)}\right|\right) \Rightarrow \int_e^{e^2}\frac{1}{x\ln(x)}\,dx = \left[\ln|\ln(x)|\right]_e^{e^2} = \ln\left|\ln(e^2)\right|-\ln|\ln(e)| = \ln(2)-\ln(1) = \ln(2).$$

9c $$\frac{x^2}{e^{2x^3}}\,dx = e^{\boxed{-2x^3}}\cdot x^2\,dx = -\frac{1}{6}\cdot e^{\boxed{-2x^3}}\,d(\boxed{-2x^3}) = d\left(-\frac{1}{6}\cdot e^{-2x^3}\right).$$

$$\int_0^1\frac{x^2}{e^{2x^3}}\,dx = \left[-\frac{1}{6}\cdot e^{-2x^3}\right]_0^1 = -\frac{1}{6}\cdot e^{-2}--\frac{1}{6}\cdot e^0 = -\frac{1}{6}\cdot\frac{1}{e^2}+\frac{1}{6} = -\frac{1}{6e^2}+\frac{1}{6}\ \text{(eventueel nog)} = -\frac{1}{6e^2}+\frac{e^2}{6e^2} = \frac{e^2-1}{6e^2}.$$

10a $$\cos(2x)\cdot\cos(x)\,dx = \left(1-2\left(\boxed{\sin(x)}\right)^2\right)d(\boxed{\sin(x)}) = d\left(\sin(x)-\frac{2}{3}\sin(x)^3\right) \Rightarrow F(x) = \sin(x)-\frac{2}{3}\sin^3(x)+c.$$

10b $\sin^5(x)\,dx = \sin^4(x)\cdot\sin(x)\,dx = -\left(\sin^2(x)\right)^2 d\left(\boxed{\cos(x)}\right) = -\left(1-\cos^2(x)\right)^2 d\left(\boxed{\cos(x)}\right) = -\left(1-\left(\boxed{\cos(x)}\right)^2\right)^2 d\left(\boxed{\cos(x)}\right)$

$= -\left(1-2\left(\boxed{\cos(x)}\right)^2+\left(\boxed{\cos(x)}\right)^4\right) d\left(\boxed{\cos(x)}\right) = \left(-1+2\left(\boxed{\cos(x)}\right)^2-\left(\boxed{\cos(x)}\right)^4\right) d\left(\boxed{\cos(x)}\right)$

$= d\left(-\cos(x)+\frac{2}{3}(\cos(x))^3-\frac{1}{5}(\cos(x))^5\right).$ Dus $G(x) = -\cos(x)+\frac{2}{3}\cos^3(x)-\frac{1}{5}\cos^5(x)+c.$

11a $\frac{2+\boxed{\ln(x)}}{x}\,dx = \left(2+\boxed{\ln(x)}\right)\cdot\frac{1}{x}\,dx = \left(2+\boxed{\ln(x)}\right) d\left(\boxed{\ln(x)}\right) = d\left(2\ln(x)+\frac{1}{2}(\ln(x))^2\right).$

$f(x) = 0$ (teller = 0) $\Rightarrow 2+\ln(x) = 0 \Rightarrow \ln(x) = -2 \Rightarrow x = e^{-2} = \frac{1}{e^2}$. Nu is $O(V) = \int_{e^{-2}}^{e} \frac{2+\ln(x)}{x}\,dx = \left[2\ln(x)+\frac{1}{2}\ln^2(x)\right]_{e^{-2}}^{e}.$

Dus $O(V) = 2\ln(e^1)+\frac{1}{2}\ln^2(e^1)-\left(2\ln(e^{-2})+\frac{1}{2}\ln^2(e^{-2})\right) = 2\cdot 1+\frac{1}{2}\cdot 1^2-\left(2\cdot -2+\frac{1}{2}\cdot(-2)^2\right) = 2+\frac{1}{2}+4-2 = 4\frac{1}{2}.$

11b $O(W) = \int_1^p \frac{2+\ln(x)}{x}\,dx = \left[2\ln(x)+\frac{1}{2}\ln^2(x)\right]_1^p = 2\ln(p)+\frac{1}{2}\ln^2(p)-\left(2\ln(1)+\frac{1}{2}\ln^2(1)\right) = 2\ln(p)+\frac{1}{2}\ln^2(p).$

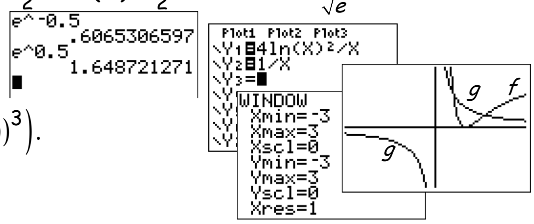
$O(W) = 6 \Rightarrow 2\ln(p)+\frac{1}{2}\ln^2(p) = 6$ (stel $\ln(p) = t$) $\Rightarrow \frac{1}{2}t^2+2t = 6 \Rightarrow t^2+4t-12 = 0 \Rightarrow (t+6)(t-2) = 0 \Rightarrow t = -6 \vee t = 2.$

$t = \ln(p) = -6 \Rightarrow p = e^{-6} = \frac{1}{e^6} < 1$ (vold. niet) en $t = \ln(p) = 2 \Rightarrow p = e^2 > 1$ (voldoet).

12a $f(x) = g(x) \Rightarrow \frac{4\ln^2(x)}{x} = \frac{1}{x} \Rightarrow 4\ln^2(x) = 1 \Rightarrow \ln^2(x) = \frac{1}{4} \Rightarrow \ln(x) = -\frac{1}{2} \vee \ln(x) = \frac{1}{2} \Rightarrow x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \vee x = e^{\frac{1}{2}} = \sqrt{e}.$

Beginvoorwaarde: $x > 0$ (vanwege f) en $x \neq 0$ (vanwege g) $\Rightarrow x > 0.$

$f(x) \leq g(x)$ (zie ook een plot) $\Rightarrow \frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}.$



12b $\frac{4\ln^2(x)}{x}\,dx = 4\left(\boxed{\ln(x)}\right)^2\cdot\frac{1}{x}\,dx = 4\left(\boxed{\ln(x)}\right)^2 d\left(\boxed{\ln(x)}\right) = d\left(\frac{4}{3}(\ln(x))^3\right).$

$O(V) = \int_{e^{-\frac{1}{2}}}^{e^{\frac{1}{2}}} \left(\frac{1}{x}-\frac{4\ln^2(x)}{x}\right)dx = \left[\ln(x)-\frac{4}{3}\ln^3(x)\right]_{e^{-\frac{1}{2}}}^{e^{\frac{1}{2}}}$

$= \ln(e^{\frac{1}{2}})-\frac{4}{3}\ln^3(e^{\frac{1}{2}})-\left(\ln(e^{-\frac{1}{2}})-\frac{4}{3}\ln^3(e^{-\frac{1}{2}})\right) = \frac{1}{2}-\frac{4}{3}\cdot(\frac{1}{2})^3 - -\frac{1}{2}+\frac{4}{3}\cdot(-\frac{1}{2})^3 = \frac{1}{2}-\frac{1}{6}+\frac{1}{2}-\frac{1}{6} = \frac{2}{3}.$

13a Beginvoorwaarde: $x^3+4 \neq 0 \Rightarrow x^3 \neq -4 \Rightarrow x \neq \sqrt[3]{-4}$. Dus V.A.: $x = \sqrt[3]{-4}$.

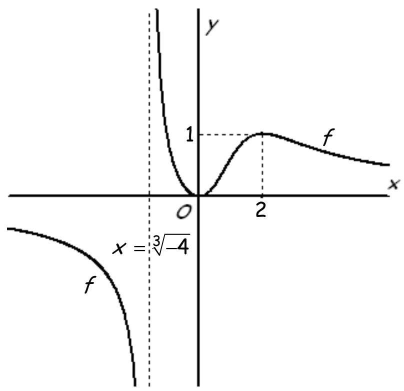
$f(x) = \frac{3x^2}{x^3+4} \Rightarrow f'(x) = \frac{(x^3+4)\cdot 6x-3x^2\cdot 3x^2}{(x^3+4)^2} = \frac{6x^4+24x-9x^4}{(x^3+4)^2} = \frac{-3x^4+24x}{(x^3+4)^2}.$

$f'(x) = 0 \Rightarrow \frac{-3x^4+24x}{(x^3+4)^2} = 0$ (teller = 0) $\Rightarrow -3x^4+24x = 0 \Rightarrow$

$-3x\cdot(x^3-8) = 0 \Rightarrow x = 0 \vee x^3 = 8 \Rightarrow x = 0 \vee x = 2.$

minimium (zie plot) is $f(0) = \frac{0}{4} = 0$ en maximium (zie plot) is $f(2) = \frac{12}{12} = 1.$

$f(x) = p$ heeft precies drie oplossingen (zie plot) voor $0 < p < 1$.



13b $\frac{3x^2}{\boxed{x^3+4}}\,dx = \frac{1}{\boxed{x^3+4}}\cdot 3x^2\,dx = \frac{1}{\boxed{x^3+4}}\,d\boxed{x^3+4} = d\ln\left|x^3+4\right|.$

$O(V) = \int_0^p \frac{3x^2}{x^3+4}\,dx = \left[\ln(x^3+4)\right]_0^p = \ln(p^3+4)-\ln(4) = \ln(\frac{p^3+4}{4}).$

$O(V) = 2 \Rightarrow \ln(\frac{p^3+4}{4}) = 2 \Rightarrow \frac{p^3+4}{4} = e^2 \Rightarrow p^3+4 = 4e^2 \Rightarrow p^3 = 4e^2-4 \Rightarrow p = \sqrt[3]{4e^2-4}.$

14a $[f(x)\cdot g(x)]' = f'(x)\cdot g(x)+f(x)\cdot g'(x)$ dus
$d(f(x)\cdot g(x)) = df(x)\cdot g(x)+f(x)\cdot dg(x)$
$= g(x)\,df(x)+f(x)\,dg(x).$

14b $d(f(x)\cdot g(x)) = g(x)\,df(x)+f(x)\,dg(x)$
$-g(x)\,df(x) = -d(f(x)\cdot g(x))+f(x)\,dg(x)$
$g(x)\,df(x) = d(f(x)\cdot g(x))-f(x)\,dg(x).$

14c $g(x) = \ln(x)$ en $f(x) = \frac{1}{2}x^2$ invullen in
$g(x)\,df(x) = d(f(x)\cdot g(x))-f(x)\,dg(x)$
geeft $\ln(x)\,d\frac{1}{2}x^2 = d\left(\frac{1}{2}x^2\cdot\ln(x)\right)-\frac{1}{2}x^2\,d\ln(x).$

14d Herleiden van het rechterlid geeft
$\ln(x)\,d\frac{1}{2}x^2 = d\left(\frac{1}{2}x^2\cdot\ln(x)\right)-\frac{1}{2}x^2\cdot\frac{1}{x}\,dx$
$= d\left(\frac{1}{2}x^2\cdot\ln(x)\right)-\frac{1}{2}x\,dx.$

14e $\ln(x)\,\mathrm{d}\frac{1}{2}x^2 = \mathrm{d}\left(\frac{1}{2}x^2 \cdot \ln(x)\right) - \frac{1}{2}x\,\mathrm{d}x$ verder herleiden geeft

$\ln(x)\,\mathrm{d}\frac{1}{2}x^2 = \mathrm{d}\left(\frac{1}{2}x^2 \cdot \ln(x)\right) - \mathrm{d}\frac{1}{4}x^2$

$= \mathrm{d}\left(\frac{1}{2}x^2 \cdot \ln(x) - \frac{1}{4}x^2\right).$

14f Dus $H(x) = \frac{1}{2}x^2 \cdot \ln(x) - \frac{1}{4}x^2$ is een primitieve van $h(x) = x\ln(x)$.

15 $x\sin(x)\,\mathrm{d}x = \sin(x)\,\mathrm{d}\frac{1}{2}x^2 = \mathrm{d}\left(\frac{1}{2}x^2 \cdot \sin(x)\right) - \frac{1}{2}x^2\,\mathrm{d}\sin(x)$

$= \mathrm{d}\left(\frac{1}{2}x^2 \cdot \sin(x)\right) - \frac{1}{2}x^2\cos(x)\,\mathrm{d}x.$

$f'g\,\mathrm{d}x = g\,\mathrm{d}f = \mathrm{d}fg - f\,\mathrm{d}g$

$\frac{1}{2}x^2\cos(x)$ is nog ingewikkelder dan de oorspronkelijke $x\sin(x)$, dus deze methode werkt niet.

16a $xe^{2x}\,\mathrm{d}x = x\,\mathrm{d}\frac{1}{2}e^{2x} = \mathrm{d}\left(\frac{1}{2}xe^{2x}\right) - \frac{1}{2}e^{2x}\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}xe^{2x}\right) - \mathrm{d}\frac{1}{4}e^{2x} = \mathrm{d}\left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) \Rightarrow F(x) = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c.$

16b $2x\cos(x)\,\mathrm{d}x = 2x\,\mathrm{d}\sin(x) = \mathrm{d}(2x\sin(x)) - \sin(x)\,\mathrm{d}2x = \mathrm{d}(2x\sin(x)) - 2\sin(x)\,\mathrm{d}x$

$= \mathrm{d}(2x\sin(x)) + \mathrm{d}2\cos(x) = \mathrm{d}(2x\sin(x) + 2\cos(x)).$ Dus $F(x) = 2x\sin(x) + 2\cos(x) + c.$

16c $x\ln^3(x)\,\mathrm{d}x = \ln^3(x)\,\mathrm{d}\frac{1}{2}x^2 = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x)\right) - \frac{1}{2}x^2\,\mathrm{d}\ln^3(x)$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x)\right) - \frac{1}{2}x^2 \cdot 3\ln^2(x) \cdot \frac{1}{x}\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x)\right) - \frac{3}{2}x \cdot \ln^2(x)\,\mathrm{d}x$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x)\right) - \ln^2(x)\,\mathrm{d}\frac{3}{4}x^2 = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x)\right) - \mathrm{d}\frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\,\mathrm{d}\ln^2(x)$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x)\right) + \frac{3}{4}x^2 \cdot 2\ln(x) \cdot \frac{1}{x}\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x)\right) + \frac{3}{2}x\ln(x)\,\mathrm{d}x$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x)\right) + \ln(x)\,\mathrm{d}\frac{3}{4}x^2 = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x)\right) + \mathrm{d}\frac{3}{4}x^2\ln(x) - \frac{3}{4}x^2\,\mathrm{d}\ln(x)$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x)\right) - \frac{3}{4}x^2 \cdot \frac{1}{x}\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x)\right) - \frac{3}{4}x\,\mathrm{d}x$

$= \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x)\right) - \mathrm{d}\frac{3}{8}x^2 = \mathrm{d}\left(\frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x) - \frac{3}{8}x^2\right).$

Dus $F(x) = \frac{1}{2}x^2\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x) - \frac{3}{8}x^2 + c.$

16d $x^3\ln(x)\,\mathrm{d}x = \ln(x)\,\mathrm{d}\frac{1}{4}x^4 = \mathrm{d}\left(\frac{1}{4}x^4\ln(x)\right) - \frac{1}{4}x^4\,\mathrm{d}\ln(x) = \mathrm{d}\left(\frac{1}{4}x^4\ln(x)\right) - \frac{1}{4}x^4 \cdot \frac{1}{x}\,\mathrm{d}x$

$= \mathrm{d}\left(\frac{1}{4}x^4\ln(x)\right) - \frac{1}{4}x^3\,\mathrm{d}x = \mathrm{d}\frac{1}{4}x^4\ln(x) - \mathrm{d}\frac{1}{16}x^4 = \mathrm{d}\left(\frac{1}{4}x^4\ln(x) - \frac{1}{16}x^4\right).$ Dus $F(x) = \frac{1}{4}x^4\ln(x) - \frac{1}{16}x^4 + c.$

17a $2x(e^x + 1)\,\mathrm{d}x = (2xe^x + 2x)\,\mathrm{d}x = 2xe^x\,\mathrm{d}x + 2x\,\mathrm{d}x = 2x\,\mathrm{d}e^x + \mathrm{d}x^2 = \mathrm{d}\left(2xe^x\right) - e^x\,\mathrm{d}2x + \mathrm{d}x^2$

$= \mathrm{d}\left(2xe^x\right) - 2e^x\,\mathrm{d}x + \mathrm{d}x^2 = \mathrm{d}\left(2xe^x\right) - \mathrm{d}2e^x + \mathrm{d}x^2 = \mathrm{d}\left(2xe^x - 2e^x + x^2\right).$

$\int_0^1 2x(e^x + 1)\,\mathrm{d}x = \left[2xe^x - 2e^x + x^2\right]_0^1 = 2e - 2e + 1 - \left(0 - 2e^0 + 0\right) = 1 + 2 = 3.$

17b $(3x+1)\sin(x)\,\mathrm{d}x = (3x+1)\,\mathrm{d}(-\cos(x)) = \mathrm{d}(-(3x+1)\cos(x)) - -\cos(x)\,\mathrm{d}(3x+1) = \mathrm{d}(-(3x+1)\cos(x)) + 3\cos(x)\,\mathrm{d}x$

$= \mathrm{d}(-(3x+1)\cos(x)) + \mathrm{d}3\sin(x) = \mathrm{d}(-(3x+1)\cos(x) + 3\sin(x)).$

$\int_0^{\pi} (3x+1)\sin(x)\,\mathrm{d}x = \left[-(3x+1)\cos(x) + 3\sin(x)\right]_0^{\pi} = -(3\pi+1)\cos(\pi) + 3\sin(\pi) - \left(-(0+1)\cdot\cos(0) + 3\sin(0)\right)$

$= -(3\pi+1)\cdot -1 + 0 - (-1\cdot 1 + 0) = 3\pi + 1 + 1 = 3\pi + 2.$

18 $\ln(x)\,\mathrm{d}x = \mathrm{d}(x\ln(x)) - x\,\mathrm{d}\ln(x) = \mathrm{d}(x\ln(x)) - x \cdot \frac{1}{x}\,\mathrm{d}x = \mathrm{d}(x\ln(x)) - 1\,\mathrm{d}x = \mathrm{d}(x\ln(x)) - \mathrm{d}x = \mathrm{d}(x\ln(x) - x).$

Dus een primitieve van $f(x) = \ln(x)$ is $F(x) = x\ln(x) - x$.

19a $f(x) = x^2\ln(x)$ BV: $x > 0 \Rightarrow f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x\ln(x) + x.$

$f'(x) = 0 \Rightarrow 2x\ln(x) + x = 0 \Rightarrow x \cdot (2\ln(x) + 1) = 0 \Rightarrow x = 0$ (vold. niet) $\vee\ \ln(x) = -\frac{1}{2} \Rightarrow x = e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}.$

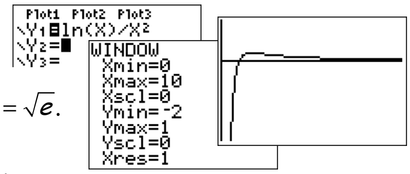
$f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \ln(e^{-\frac{1}{2}}) = e^{-1} \cdot -\frac{1}{2} = -\frac{1}{2} \cdot \frac{1}{e} = -\frac{1}{2e} \Rightarrow A(\frac{1}{\sqrt{e}}, -\frac{1}{2e}).$

19b $x^2\ln(x)\,\mathrm{d}x = \ln(x)\,\mathrm{d}\frac{1}{3}x^3 = \mathrm{d}\left(\frac{1}{3}x^3\ln(x)\right) - \frac{1}{3}x^3\,\mathrm{d}\ln(x) = \mathrm{d}\left(\frac{1}{3}x^3\ln(x)\right) - \frac{1}{3}x^3 \cdot \frac{1}{x}\,\mathrm{d}x$

$= \mathrm{d}\left(\frac{1}{3}x^3\ln(x)\right) - \frac{1}{3}x^2\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{3}x^3\ln(x)\right) - \mathrm{d}\frac{1}{9}x^3 = \mathrm{d}\left(\frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3\right).$

$O(V) = \int_1^e f(x)\,\mathrm{d}x = \left[\frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3\right]_1^e = \frac{1}{3}e^3\ln(e) - \frac{1}{9}e^3 - \left(\frac{1}{3}\ln(1) - \frac{1}{9}\right) = \frac{1}{3}e^3 - \frac{1}{9}e^3 - (0 - \frac{1}{9}) = \frac{2}{9}e^3 + \frac{1}{9}.$

20a $f(x)=\frac{\ln(x)}{x^2}$ BV: $x>0 \Rightarrow f'(x)=\frac{x^2\cdot\frac{1}{x}-\ln(x)\cdot 2x}{(x^2)^2}=\frac{x-2x\ln(x)}{x^4}=\frac{1-2\ln(x)}{x^3}$.



$f'(x)=0$ (teller = 0) $\Rightarrow 1-2\ln(x)=0 \Rightarrow -2\ln(x)=-1 \Rightarrow \ln(x)=\frac{1}{2} \Rightarrow x=e^{\frac{1}{2}}=\sqrt{e}$.

maximum (zie plot) is $f(\sqrt{e})=\frac{\ln(e^{\frac{1}{2}})}{e}=\frac{\frac{1}{2}}{e}=\frac{1}{2}\cdot\frac{1}{e}=\frac{1}{2e} \Rightarrow B_f$ (zie plot) $=\left\langle \leftarrow, \frac{1}{2e}\right]$.

20b $\frac{\ln(x)}{x^2}\,dx=\ln(x)\cdot x^{-2}\,dx=\ln(x)\,d\left(-x^{-1}\right)=\ln(x)\,d\frac{-1}{x}=d\left(-\frac{1}{x}\ln(x)\right)--\frac{1}{x}\,d\ln(x)=d\left(-\frac{\ln(x)}{x}\right)+\frac{1}{x}\cdot\frac{1}{x}\,dx$

$=d\left(-\frac{\ln(x)}{x}\right)+x^{-2}\,dx=d\left(-\frac{\ln(x)}{x}\right)-dx^{-1}=d\left(-\frac{\ln(x)}{x}-\frac{1}{x}\right)=d\left(\frac{-\ln(x)-1}{x}\right)$.

$f(x)=0$ (teller = 0) $\Rightarrow \ln(x)=0 \Rightarrow x=1$.

$O(V)=\int_1^e f(x)\,dx=\left[\frac{-\ln(x)-1}{x}\right]_1^e=\frac{-\ln(e)-1}{e}-\left(\frac{-\ln(1)-1}{1}\right)=\frac{-1-1}{e}-\left(\frac{-0-1}{1}\right)=-\frac{2}{e}+1$.

21a $x^2e^x\,dx=x^2\,de^x=dx^2e^x-e^x\,dx^2=dx^2e^x-2xe^x\,dx$.

21b $2xe^x\,dx=2x\,de^x=d2xe^x-e^x\,d2x=d2xe^x-2e^x\,dx=d2xe^x-d2e^x=d(2xe^x-2e^x)$.

21c $x^2e^x\,dx=dx^2e^x-2xe^x\,dx$ (zie 21a) $=dx^2e^x-d(2xe^x-2e^x)$ (zie 21b) $=d(x^2e^x-2xe^x+2e^x)$.

Dus $F(x)=x^2e^x-2xe^x+2e^x+c=(x^2-2x+2)\cdot e^x+c$.

22 $e^x\sin(x)dx=\sin(x)de^x=de^x\sin(x)-e^x\,d\sin(x)=de^x\sin(x)-e^x\cos(x)dx=de^x\sin(x)-\cos(x)de^x$

$=de^x\sin(x)-de^x\cos(x)+e^x\,d\cos(x)=d\left(e^x\sin(x)-e^x\cos(x)\right)-e^x\sin(x)dx$.

Uit $e^x\sin(x)dx=d\left(e^x\sin(x)-e^x\cos(x)\right)-e^x\sin(x)dx$ volgt nu

$2e^x\sin(x)dx=d\left(e^x\sin(x)-e^x\cos(x)\right) \Rightarrow e^x\sin(x)dx=d\left(\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)\right)$.

Dus $G(x)=\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)+c$.

23a $\frac{1}{4}x^2\cos(x)dx=\frac{1}{4}x^2\,d\sin(x)=d\frac{1}{4}x^2\sin(x)-\sin(x)d\frac{1}{4}x^2=d\frac{1}{4}x^2\sin(x)-\frac{1}{2}x\sin(x)dx$

$=d\frac{1}{4}x^2\sin(x)+\frac{1}{2}x\,d\cos(x)=d\frac{1}{4}x^2\sin(x)+d\frac{1}{2}x\cos(x)-\cos(x)d\frac{1}{2}x$

$=d\frac{1}{4}x^2\sin(x)+d\frac{1}{2}x\cos(x)-\frac{1}{2}\cos(x)dx=d\frac{1}{4}x^2\sin(x)+d\frac{1}{2}x\cos(x)-d\frac{1}{2}\sin(x)$

$=d\left(\frac{1}{4}x^2\sin(x)+\frac{1}{2}x\cos(x)-\frac{1}{2}\sin(x)\right)$. Dus $F(x)=\frac{1}{4}x^2\sin(x)+\frac{1}{2}x\cos(x)-\frac{1}{2}\sin(x)+c$.

23b $e^{-x}\cos(x)dx=e^{-x}\,d\sin(x)=de^{-x}\sin(x)-\sin(x)de^{-x}=de^{-x}\sin(x)+e^{-x}\sin(x)dx=de^{-x}\sin(x)-e^{-x}\,d\cos(x)$

$=de^{-x}\sin(x)-de^{-x}\cos(x)+\cos(x)de^{-x}=d\left(e^{-x}\sin(x)-e^{-x}\cos(x)\right)-e^{-x}\cos(x)dx$.

Uit $e^{-x}\cos(x)dx=d\left(e^{-x}\sin(x)-e^{-x}\cos(x)\right)-e^{-x}\cos(x)dx$ volgt nu

$2e^{-x}\cos(x)dx=d\left(e^{-x}\sin(x)-e^{-x}\cos(x)\right) \Rightarrow e^{-x}\cos(x)dx=d\,\frac{1}{2}\left(e^{-x}\sin(x)-e^{-x}\cos(x)\right)$.

Dus $G(x)=\frac{1}{2}\left(e^{-x}\sin(x)-e^{-x}\cos(x)\right)+c=\frac{1}{2}e^{-x}\left(\sin(x)-\cos(x)\right)+c$.

23c $e^{2x}\sin(x)dx=-e^{2x}\,d\cos(x)=d\left(-e^{2x}\cos(x)\right)+\cos(x)de^{2x}=d\left(-e^{2x}\cos(x)\right)+2e^{2x}\cos(x)dx$

$=d\left(-e^{2x}\cos(x)\right)+2e^{2x}\,d\sin(x)=d\left(-e^{2x}\cos(x)\right)+d2e^{2x}\sin(x)-\sin(x)d2e^{2x}$

$=d\left(-e^{2x}\cos(x)+2e^{2x}\sin(x)\right)-4e^{2x}\sin(x)dx$.

Uit $e^{2x}\sin(x)dx=d\left(-e^{2x}\cos(x)+2e^{2x}\sin(x)\right)-4e^{2x}\sin(x)dx$ volgt nu

$5\cdot e^{2x}\sin(x)dx=d\left(-e^{2x}\cos(x)+2e^{2x}\sin(x)\right) \Rightarrow e^{2x}\sin(x)dx=d\,\frac{1}{5}\left(-e^{2x}\cos(x)+2e^{2x}\sin(x)\right)$.

Dus $H(x)=\frac{1}{5}\left(-e^{2x}\cos(x)+2e^{2x}\sin(x)\right)+c=\frac{1}{5}e^{2x}\left(-\cos(x)+2\sin(x)\right)+c$.

24a $(x^2-x)e^x\,dx=(x^2-x)\,de^x=d(x^2-x)e^x-e^x\,d(x^2-x)=d(x^2-x)e^x-(2x-1)e^x\,dx$

$=d(x^2-x)e^x-(2x-1)\,de^x=d(x^2-x)e^x-d(2x-1)e^x+e^x\,d(2x-1)$

$=d(x^2-x)e^x-d(2x-1)e^x+d2e^x=d\left((x^2-x)e^x-(2x-1)e^x+2e^x\right)=d\left((x^2-3x+3)e^x\right)$.

$\int_1^3 (x^2-x)e^x\,dx=\left[(x^2-3x+3)e^x\right]_1^3=(9-9+3)e^3-(1-3+3)e=3e^3-e$.

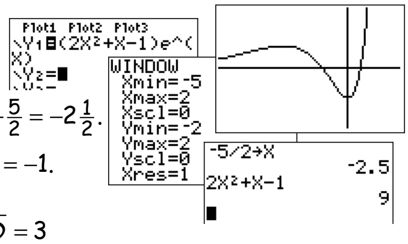
24b $x\ln^2(x)\,dx = \ln^2(x)\,d\tfrac{1}{2}x^2 = d\left(\tfrac{1}{2}x^2\ln^2(x)\right) - \tfrac{1}{2}x^2\,d\ln^2(x) = d\left(\tfrac{1}{2}x^2\ln^2(x)\right) - \tfrac{1}{2}x^2\cdot 2\ln(x)\cdot\tfrac{1}{x}\,dx$

$= d\left(\tfrac{1}{2}x^2\cdot\ln^2(x)\right) - x\ln(x)\,dx = d\left(\tfrac{1}{2}x^2\cdot\ln^2(x)\right) - \ln(x)\,d\tfrac{1}{2}x^2 = \left(d\tfrac{1}{2}x^2\cdot\ln^2(x)\right) - d\tfrac{1}{2}x^2\cdot\ln(x) + \tfrac{1}{2}x^2 d\ln(x)$

$= d\left(\tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x)\right) + \tfrac{1}{2}x^2\cdot\tfrac{1}{x}\,dx = d\left(\tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x)\right) + \tfrac{1}{2}x\,dx$

$= d\left(\tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x)\right) + d\tfrac{1}{4}x^2 = d\left(\tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \tfrac{1}{4}x^2\right).$

$\int_1^e x\ln^2(x)\,dx = \left[\tfrac{1}{2}x^2\cdot\ln^2(x) - \tfrac{1}{2}x^2\cdot\ln(x) + \tfrac{1}{4}x^2\right]_1^e = \tfrac{1}{2}e^2\cdot 1^2 - \tfrac{1}{2}e^2\cdot 1 + \tfrac{1}{4}e^2 - (\tfrac{1}{2}\cdot 0^2 - \tfrac{1}{2}\cdot 0 + \tfrac{1}{4}) = \tfrac{1}{4}e^2 - \tfrac{1}{4}.$



25a $f(x) = (2x^2 + x - 1)e^x \Rightarrow f'(x) = (4x+1)e^x + (2x^2+x-1)e^x = (2x^2+5x)e^x.$

$f'(x) = 0 \Rightarrow (2x^2+5x)e^x = 0 \Rightarrow 2x^2+5x = 0 \Rightarrow x(2x+5) = 0 \Rightarrow x = 0 \vee x = -\tfrac{5}{2} = -2\tfrac{1}{2}.$

max. (zie plot) is $f(-2\tfrac{1}{2}) = 9\cdot e^{-2\frac{1}{2}} = \frac{9}{e^{2\frac{1}{2}}} = \frac{9}{e^2\cdot\sqrt{e}}$ en min. (zie plot) is $f(0) = -1\cdot e^0 = -1.$

25b $f(x) = 0 \Rightarrow (2x^2+x-1)e^x = 0 \Rightarrow 2x^2+x-1 = 0$ met $D = 1^2 - 4\cdot 2\cdot -1 = 9 \Rightarrow \sqrt{D} = 3$

$x = \frac{-1+3}{2\cdot 2} = \frac{2}{4} = \frac{1}{2} \vee x = \frac{-1-3}{4} = \frac{-4}{4} = -1.$

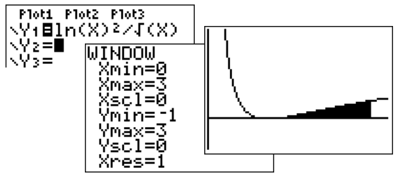
$(2x^2+x-1)e^x\,dx = (2x^2+x-1)\,de^x = d(2x^2+x-1)e^x - e^x\,d(2x^2+x-1) = d(2x^2+x-1)e^x - (4x+1)e^x\,dx$

$= d(2x^2+x-1)e^x - (4x+1)\,de^x = d(2x^2+x-1)e^x - d(4x+1)e^x + e^x\,d(4x+1)$

$= d(2x^2+x-1)e^x - d(4x+1)e^x + 4e^x\,dx = d(2x^2+x-1)e^x - d(4x+1)e^x + d\,4e^x$

$= d(2x^2+x-1-4x-1+4)e^x = d(2x^2-3x+2)e^x.$

$O(V) = \int_{-1}^{\frac{1}{2}}(0 - f(x))\,dx = \int_{-1}^{\frac{1}{2}} -f(x)\,dx = \left[-(2x^2-3x+2)e^x\right]_{-1}^{\frac{1}{2}} = -(\tfrac{2}{4} - \tfrac{3}{2} + 2)e^{\frac{1}{2}} + (2+3+2)e^{-1} = -e^{\frac{1}{2}} + 7\cdot\tfrac{1}{e} = \tfrac{7}{e} - \sqrt{e}.$



26a $f(x) = \frac{\ln^2(x)}{\sqrt{x}} = 0$ (teller = 0) BV: $x > 0 \Rightarrow \ln^2(x) = 0 \Rightarrow \ln(x) = 0 \Rightarrow x = 1.$

$\frac{\ln^2(x)}{\sqrt{x}}\,dx = \ln^2(x)\cdot\frac{1}{\sqrt{x}}\,dx = \ln^2(x)\,d2\sqrt{x} = d2\sqrt{x}\cdot\ln^2(x) - 2\sqrt{x}\,d\ln^2(x)$

$= d2\sqrt{x}\cdot\ln^2(x) - 2\sqrt{x}\cdot 2\ln(x)\cdot\frac{1}{x}\,dx = d2\sqrt{x}\cdot\ln^2(x) - \frac{4\ln(x)}{\sqrt{x}}\,dx$

$= d2\sqrt{x}\cdot\ln^2(x) - 4\ln(x)\,d2\sqrt{x} = d2\sqrt{x}\cdot\ln^2(x) - d8\sqrt{x}\cdot\ln(x) + 2\sqrt{x}\,d4\ln(x)$

$= d2\sqrt{x}\cdot\ln^2(x) - d8\sqrt{x}\cdot\ln(x) + 2\sqrt{x}\cdot 4\cdot\frac{1}{x}\,dx = d2\sqrt{x}\cdot\ln^2(x) - d8\sqrt{x}\cdot\ln(x) + \frac{8}{\sqrt{x}}\,dx$

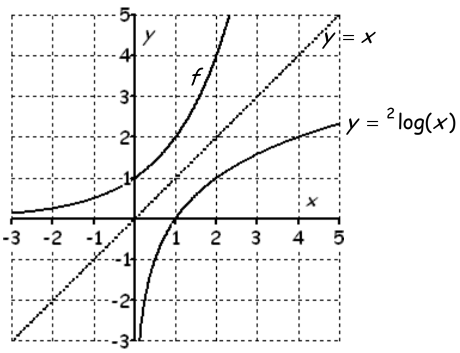
$= d2\sqrt{x}\cdot\ln^2(x) - d8\sqrt{x}\cdot\ln(x) + d16\sqrt{x} = d\left(2\sqrt{x}\cdot\ln^2(x) - 8\sqrt{x}\cdot\ln(x) + 16\sqrt{x}\right).$

$O(V) = \int_1^e f(x)\,dx = \left[2\sqrt{x}\cdot\ln^2(x) - 8\sqrt{x}\cdot\ln(x) + 16\sqrt{x}\right]_1^e = 2\sqrt{e}\cdot 1^2 - 8\sqrt{e}\cdot 1 + 16\sqrt{e} - (2\cdot 0^2 - 8\cdot 0 + 16) = 10\sqrt{e} - 16.$

26b $I(L) = \int_1^e \pi\cdot(f(x))^2\,dx = \int_1^e \pi\cdot\frac{\ln^4(x)}{x}\,dx = \left[\tfrac{1}{5}\ln^5(x)\right]_1^e = \tfrac{1}{5}\ln^5(e) - \tfrac{1}{5}\ln^5(1) = \tfrac{1}{5}\cdot 1^5 - \tfrac{1}{5}\cdot 0^5 = \tfrac{1}{5}.$

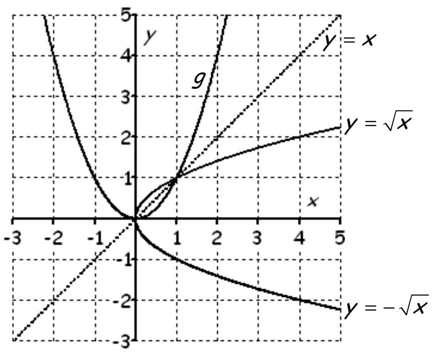
Immers: $\frac{\ln^4(x)}{x}\,dx = \ln^4(x)\,d\ln(x) = d\,\tfrac{1}{5}\ln^5(x).$

27ab



Het spiegelbeeld is de grafiek van een functie.

27cd



Het spiegelbeeld is niet de grafiek van een functie.
(want bijvoorbeeld $x = 4$ heeft twee beelden, namelijk 2 en -2)

27e Bij het domein $[0,\rightarrow\rangle$ is het spiegelbeeld van de grafiek van $g(x) = x^2$ wel de grafiek van een functie, namelijk de functie $h(x) = \sqrt{x}$.

28a $\frac{1}{\tan^2(t)+1}\,\mathrm{d}\tan(t) = \frac{1}{\tan^2(t)+1}\cdot(1+\tan^2(t))\,\mathrm{d}t = \frac{\tan^2(t)+1}{\tan^2(t)+1}\,\mathrm{d}t = 1\,\mathrm{d}t = \mathrm{d}t.$

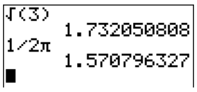
Want $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} \Rightarrow f'(x) = \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot -\sin(x)}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = 1 + \frac{\sin^2(x)}{\cos^2(x)} = 1+\tan^2(x).$

28b $x = \tan(t)$ ofwel $t = \tan^{-1}(x)$ en $F(x) = \tan^{-1}(x) \Rightarrow F'(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x} = \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{1}{1+\tan^2(t)} = \frac{1}{1+x^2} = \frac{1}{x^2+1}.$

29a Het bereik (de uitkomsten) van $f(x) = \arctan(x)$ is (liggen in) $\left\langle -\frac{1}{2}\pi,\ \frac{1}{2}\pi \right\rangle$.

$1\frac{1}{6}\pi$ ligt niet in $\left\langle -\frac{1}{2}\pi,\ \frac{1}{2}\pi \right\rangle \Rightarrow \arctan(\frac{1}{3}\sqrt{3}) \neq 1\frac{1}{6}\pi.$

29b $\sqrt{3} > \frac{1}{2}\pi \Rightarrow \arctan(x) = \sqrt{3}$ heeft geen oplossing.



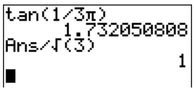
α	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\sin(\alpha)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\alpha)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\tan(\alpha)$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	↯

30 Zie de tabel hiernaast.

x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
$\arctan(x)$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

31a $\arctan(x) = \frac{1}{3}\pi$

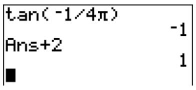
$x = \tan(\frac{1}{3}\pi) = \sqrt{3}.$



31b $\arctan(x-2) = -\frac{1}{4}\pi$

$x - 2 = \tan(-\frac{1}{4}\pi) = -1$

$x = -1+2 = 1.$

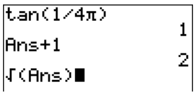


31c $\arctan(x^2-1) = \frac{1}{4}\pi$

$x^2 - 1 = \tan(\frac{1}{4}\pi) = 1$

$x^2 = 2$

$x = -\sqrt{2} \vee x = \sqrt{2}.$

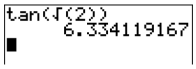


31d $\arctan(x) = \frac{2}{3}\pi$

geen oplossing want $\frac{2}{3}\pi > \frac{1}{2}\pi$ ($\frac{2}{3} > \frac{1}{2}$).

31e $\arctan(x) = \sqrt{2}$

$x = \tan(\sqrt{2}) \approx 6{,}334.$

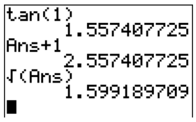


31f $\arctan(x^2-1) = 1$

$x^2 - 1 = \tan(1) \approx 1{,}5574$

$x^2 \approx 2{,}5574$

$x \approx -1{,}599 \vee x \approx 1{,}599.$



32a $f(x) = 2\arctan(\boxed{\frac{1}{2}x}) \Rightarrow f'(x) = 2\cdot\frac{1}{(\frac{1}{2}x)^2+1}\cdot\frac{1}{2} = \frac{1}{\frac{1}{4}x^2+1} = \frac{1}{\frac{1}{4}x^2+1}\cdot\frac{4}{4} = \frac{4}{x^2+4}.$

32b $g(x) = \arctan(\boxed{x-2}) \Rightarrow g'(x) = \frac{1}{(x-2)^2+1}\cdot 1 = \frac{1}{(x-2)^2+1}.$

32c $h(x) = \arctan(\boxed{x^2}) \Rightarrow h'(x) = \frac{1}{(x^2)^2+1}\cdot 2x = \frac{2x}{x^4+1}.$

33a $\int_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2+1}\,\mathrm{d}x = \left[\arctan(x)\right]_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(-\frac{1}{3}\sqrt{3}) = \frac{1}{3}\pi - -\frac{1}{6}\pi = \frac{2}{6}\pi + \frac{1}{6}\pi = \frac{1}{2}\pi.$

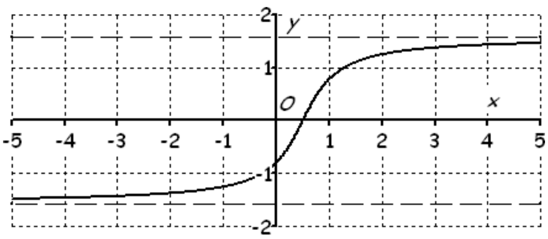
33b $\int_{-2}^{-1} \frac{1}{(x+1)^2+1}\,\mathrm{d}x = \left[\arctan(x+1)\right]_{-2}^{-1} = \arctan(0) - \arctan(-1) = 0 - -\frac{1}{4}\pi = \frac{1}{4}\pi.$

33c $\int_{0}^{\frac{1}{3}\sqrt{3}} \frac{3}{(3x)^2+1}\,\mathrm{d}x = \left[\arctan(3x)\right]_{0}^{\frac{1}{3}\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(0) = \frac{1}{3}\pi - 0 = \frac{1}{3}\pi.$

34a $y = \arctan(x) \xrightarrow{\text{1 naar rechts}} y = \arctan(x-1) \xrightarrow{\text{Verm. t.o.v. } x\text{-as } \frac{1}{2}} f(x) = \arctan(2x-1).$

De horizontale asymptoten zijn $y = -\frac{1}{2}\pi$ en $y = \frac{1}{2}\pi$.

(maak een schets van de grafiek van f en stippel de asymptoten!!!)



34b f snijden met de x-as ($y=0$) $\Rightarrow f(x) = 0 \Rightarrow \arctan(2x-1) = 0 \Rightarrow$

$2x - 1 = \tan(0) = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$. Dus $S(\frac{1}{2},\ 0)$.

$f(x) = \arctan(\boxed{2x-1}) \Rightarrow f'(x) = \frac{1}{(2x-1)^2+1}\cdot 2 = \frac{2}{(2x-1)^2+1}.$

$\left.\begin{array}{l} f'(\frac{1}{2}) = \frac{2}{(1-1)^2+1} = \frac{2}{0+1} = 2 \\ k\!:\ y = 2x + b \text{ door } S(\frac{1}{2},\ 0) \end{array}\right\} \Rightarrow 0 = 2\cdot\frac{1}{2} + b \Rightarrow -1 = b.$ Dus $k\!:\ y = 2x - 1.$

34c $\arctan(2x-1) = -\frac{1}{4}\pi$

$2x - 1 = \tan(-\frac{1}{4}\pi) = -1$

$2x = 0$

$x = 0$

en

$\arctan(2x-1) = \frac{1}{3}\pi$

$2x - 1 = \tan(\frac{1}{3}\pi) = \sqrt{3}$

$2x = 1 + \sqrt{3}$

$x = \frac{1}{2} + \frac{1}{2}\sqrt{3}.$

Dus $-\frac{1}{4}\pi < f(x) < \frac{1}{3}\pi \Rightarrow 0 < x < \frac{1}{2} + \frac{1}{2}\sqrt{3}.$

(gebruik de bereking hiernaast en een plot)

34d $f(1) = \arctan(2 \cdot 1 - 1) = \arctan(1) = \frac{1}{4}\pi$.
$x \le 1$ (gebruik de berekening hierboven, een plot en het bereik van f) $\Rightarrow -\frac{1}{2}\pi < f(x) \le \frac{1}{4}\pi$.

35a $F(x) = \frac{1}{2}\arctan(\boxed{2x}) + c \Rightarrow F'(x) = \frac{1}{2} \cdot \frac{1}{(2x)^2+1} \cdot 2 = \frac{1}{(2x)^2+1} = \frac{1}{4x^2+1} = f(x)$.

35b $G(x) = 2\arctan(\boxed{\frac{1}{2}x}) + c \Rightarrow G'(x) = 2 \cdot \frac{1}{(\frac{1}{2}x)^2+1} \cdot \frac{1}{2} = \frac{1}{(\frac{1}{2}x)^2+1} = \frac{1}{\frac{1}{4}x^2+1} = \frac{1}{\frac{1}{4}x^2+1} \cdot \frac{4}{4} = \frac{4}{x^2+4} = g(x)$.

36a $f(x)\mathrm{d}x = \frac{12}{16x^2+1}\mathrm{d}x = 12 \cdot \frac{1}{(\boxed{4x})^2+1}\mathrm{d}x = 3 \cdot \frac{1}{(\boxed{4x})^2+1}\mathrm{d}\boxed{4x} = \mathrm{d}(3\arctan(4x)) \Rightarrow F(x) = 3\arctan(4x) + c \cdot$

36b $g(x)\mathrm{d}x = \frac{4}{x^2+4}\mathrm{d}x = \frac{1}{\frac{1}{4}x^2+1}\mathrm{d}x = 2 \cdot \frac{1}{(\boxed{\frac{1}{2}x})^2+1}\mathrm{d}\boxed{\frac{1}{2}x} = \mathrm{d}\left(2\arctan(\frac{1}{2}x)\right) \Rightarrow G(x) = 2\arctan(\frac{1}{2}x) + c \cdot$

36c $h(x)\mathrm{d}x = \arctan(2x)\mathrm{d}x = \mathrm{d}(x\arctan(2x)) - x\,\mathrm{d}(\arctan(2x)) = \mathrm{d}(x\arctan(2x)) - x \cdot \frac{1}{(2x)^2+1} \cdot 2\mathrm{d}x$

$= \mathrm{d}(x\arctan(2x)) - \frac{2x}{4x^2+1}\mathrm{d}x = \mathrm{d}(x\arctan(2x)) - \frac{1}{4} \cdot \frac{8x}{4x^2+1}\mathrm{d}x = \mathrm{d}(x\arctan(2x)) - \frac{1}{4} \cdot \frac{1}{\boxed{4x^2+1}}\mathrm{d}\left(\boxed{4x^2+1}\right)$

$= \mathrm{d}(x\arctan(2x)) - \frac{1}{4} \cdot \mathrm{d}\ln(4x^2+1) = \mathrm{d}\left(x\arctan(2x) - \frac{1}{4} \cdot \ln(4x^2+1)\right)$.

Dus $H(x) = x\arctan(2x) - \frac{1}{4} \cdot \ln(4x^2+1) + c \cdot$

36d $j(x)\mathrm{d}x = \frac{3}{x^2+4x+5}\mathrm{d}x = \frac{3}{x^2+4x+4+1}\mathrm{d}x = \frac{3}{(\boxed{x+2})^2+1}\mathrm{d}x = \frac{3}{(\boxed{x+2})^2+1}\mathrm{d}\boxed{x+2} \Rightarrow J(x) = 3 \cdot \arctan(x+2) + c$.

37a $\int_0^1 \frac{3}{x^2+3}\mathrm{d}x = \int_0^1 \frac{1}{\frac{1}{3}x^2+1}\mathrm{d}x = \int_0^1 \frac{1}{(\frac{x}{\sqrt{3}})^2+1}\mathrm{d}x = \left[\sqrt{3}\arctan(\frac{x}{\sqrt{3}})\right]_0^1 = \sqrt{3}\arctan(\frac{1}{\sqrt{3}}) - \sqrt{3}\arctan(0) = \sqrt{3}\arctan(\frac{1}{3}\sqrt{3}) = \sqrt{3} \cdot \frac{1}{6}\pi$.

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

37b $\int_0^1 \frac{x}{x^4+1}\mathrm{d}x = \int_0^1 \frac{x}{(x^2)^2+1}\mathrm{d}x = \int_0^1 \frac{\frac{1}{2}}{(x^2)^2+1}\mathrm{d}x^2 = \left[\frac{1}{2}\arctan(x^2)\right]_0^1 = \frac{1}{2}\arctan(1) - \frac{1}{2}\arctan(0) = \frac{1}{2} \cdot \frac{1}{4}\pi = \frac{1}{8}\pi$.

37c $\int_3^6 \frac{5}{x^2-6x+18}\mathrm{d}x = \int_3^6 \frac{5}{(x-3)^2-9+18}\mathrm{d}x = \int_3^6 \frac{5}{(x-3)^2+9}\mathrm{d}x = \int_3^6 \frac{\frac{5}{9}}{\frac{1}{9}(x-3)^2+1}\mathrm{d}x = \int_3^6 \frac{\frac{5}{9}}{(\frac{1}{3}(x-3))^2+1}\mathrm{d}x$

$= \left[\frac{5}{9} \cdot 3\arctan(\frac{1}{3}(x-3))\right]_3^6 = \left[\frac{5}{3}\arctan(\frac{1}{3}x-1)\right]_3^6 = \frac{5}{3}\arctan(1) - \frac{5}{3}\arctan(0) = \frac{5}{3} \cdot \frac{1}{4}\pi = \frac{5}{12}\pi$.

37d $\int_0^{\sqrt{3}} \frac{\arctan(x)}{x^2+1}\mathrm{d}x = \int_0^{\sqrt{3}} \arctan(x)\mathrm{d}\arctan(x) = \int_0^{\sqrt{3}} \mathrm{d}\frac{1}{2}(\arctan(x))^2 = \left[\frac{1}{2}(\arctan(x))^2\right]_0^{\sqrt{3}}$

$= \frac{1}{2}(\arctan(\sqrt{3}))^2 - \frac{1}{2}(\arctan(0))^2 = \frac{1}{2} \cdot (\frac{1}{3}\pi)^2 - \frac{1}{2} \cdot (0)^2 = \frac{1}{2} \cdot \frac{1}{9}\pi^2 = \frac{1}{18}\pi^2$.

38a $\int_0^{1\frac{1}{2}} \frac{2}{4x^2+9}\mathrm{d}x = \int_0^{1\frac{1}{2}} \frac{\frac{2}{9}}{\frac{4}{9}x^2+1}\mathrm{d}x = \int_0^{1\frac{1}{2}} \frac{\frac{2}{9}}{(\frac{2}{3}x)^2+1}\mathrm{d}x = \left[\frac{2}{9} \cdot \frac{3}{2}\arctan(\frac{2}{3}x)\right]_0^{1\frac{1}{2}} = \frac{1}{3}\arctan(1) - \frac{1}{3}\arctan(0) = \frac{1}{3} \cdot \frac{1}{4}\pi = \frac{1}{12}\pi$.

38b $\int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x}+1}\mathrm{d}x = \int_0^{\frac{1}{2}\ln(3)} \frac{1}{(e^x)^2+1}\mathrm{d}e^x = \left[\arctan(e^x)\right]_0^{\frac{1}{2}\ln(3)} = \arctan(e^{\frac{1}{2}\ln(3)}) - \arctan(e^0) = \arctan(e^{\ln(3^{\frac{1}{2}})}) - \arctan(1)$

$= \arctan(e^{\ln(\sqrt{3})}) - \frac{1}{4}\pi = \arctan(\sqrt{3}) - \frac{1}{4}\pi = \frac{1}{3}\pi - \frac{1}{4}\pi = \frac{1}{12}\pi$.

38c $\int_0^1 \frac{1}{2x^2-2x+1}\mathrm{d}x = \int_0^1 \frac{2}{4x^2-4x+2}\mathrm{d}x = \int_0^1 \frac{2}{4x^2-4x+1+1}\mathrm{d}x = \int_0^1 \frac{2}{(2x-1)^2+1}\mathrm{d}x = \int_0^1 \frac{1}{(2x-1)^2+1}\mathrm{d}(2x-1)$

$= \left[\arctan(2x-1)\right]_0^1 = \arctan(1) - \arctan(-1) = \frac{1}{4}\pi - -\frac{1}{4}\pi = \frac{1}{2}\pi$.

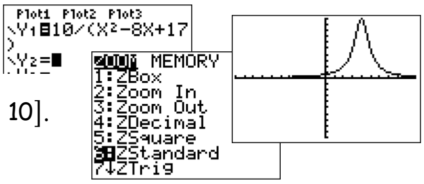
38d $\frac{\ln(x^2+1)}{x^2}\mathrm{d}x = \ln(x^2+1) \cdot \frac{1}{x^2}\mathrm{d}x = \ln(x^2+1) \cdot x^{-2}\mathrm{d}x = \ln(x^2+1)\mathrm{d}(-x^{-1}) = \mathrm{d}\left(-\frac{1}{x} \cdot \ln(x^2+1)\right) - -\frac{1}{x}\mathrm{d}\ln(x^2+1)$

$= \mathrm{d}\left(-\frac{\ln(x^2+1)}{x}\right) + \frac{1}{x} \cdot \frac{1}{x^2+1} \cdot 2x\,\mathrm{d}x = \mathrm{d}\left(-\frac{\ln(x^2+1)}{x}\right) + \frac{2}{x^2+1}\mathrm{d}x = \mathrm{d}\left(-\frac{\ln(x^2+1)}{x} + 2\arctan(x)\right)$.

$\int_1^{\sqrt{3}} \frac{\ln(x^2+1)}{x^2}\mathrm{d}x = \left[-\frac{\ln(x^2+1)}{x} + 2\arctan(x)\right]_1^{\sqrt{3}} = -\frac{\ln(4)}{\sqrt{3}} + 2\arctan(\sqrt{3}) - (-\frac{\ln(2)}{1} + 2\arctan(1))$

$= -\frac{\ln(4)}{\sqrt{3}} + 2 \cdot \frac{1}{3}\pi + \ln(2) - 2 \cdot \frac{1}{4}\pi = -\frac{\ln(4)}{\sqrt{3}} + \frac{2}{3}\pi + \ln(2) - \frac{1}{2}\pi = -\frac{\ln(4)}{\sqrt{3}} + \frac{1}{6}\pi + \ln(2)$.

39a $f(x)=\frac{10}{x^2-8x+17} \Rightarrow f'(x)=\frac{(x^2-8x+17)\cdot 0-10\cdot(2x-8)}{(x^2-8x+17)^2}=\frac{-20x+80}{(x^2-8x+17)^2}$.



$f'(x)=0$ (teller = 0) $\Rightarrow -20x+80=0 \Rightarrow -20x=-80 \Rightarrow x=4$.

Maximum (zie plot) is $f(4)=\frac{10}{4^2-8\cdot4+17}=\frac{10}{16-32+17}=\frac{10}{1}=10 \Rightarrow B_f=\langle 0,\,10]$.

39b $f(4-p)=\frac{10}{(4-p)^2-8\cdot(4-p)+17}=\frac{10}{16-8p+p^2-32+8p+17}=\frac{10}{p^2+1}$ en

$f(4+p)=\frac{10}{(4+p)^2-8\cdot(4+p)+17}=\frac{10}{16+8p+p^2-32-8p+17}=\frac{10}{p^2+1}$.

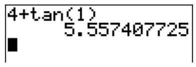
Voor elke p geldt: $f(4-p)=f(4+p) \Rightarrow f$ is symmetrisch in de lijn $x=4$.

39c $O(V)=\int_4^5 \frac{10}{x^2-8x+17}\,dx=\int_4^5 \frac{10}{(x-4)^2-16+17}\,dx=\int_4^5 \frac{10}{(x-4)^2+1}\,d(x-4)=\left[10\arctan(x-4)\right]_4^5$

$=10\arctan(1)-10\arctan(0)=10\cdot\frac{1}{4}\pi=2\frac{1}{2}\pi$.

39d $O(W)=10 \Rightarrow \int_4^p \frac{10}{x^2-8x+17}\,dx=10 \Rightarrow \left[10\arctan(x-4)\right]_4^p=10 \Rightarrow 10\arctan(p-4)-10\arctan(0)=10 \Rightarrow$

$10\arctan(p-4)-0=10 \Rightarrow \arctan(p-4)=1 \Rightarrow p-4=\tan(1) \Rightarrow p=4+\tan(1)\approx 5{,}557$.



40a Elke functiewaarde (uitkomst) van $f(x)=\sin(x)$ met $D_f=[-\frac{1}{2}\pi,\ \frac{1}{2}\pi]$ komt precies één keer voor.

40b Dit kan alleen met het domein $[0,\ \pi] \Rightarrow$ II.

41 Zie de tabel hiernaast.

x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\arccos(x)$	π	$\frac{5}{6}\pi$	$\frac{3}{4}\pi$	$\frac{2}{3}\pi$	$\frac{1}{2}\pi$	$\frac{1}{3}\pi$	$\frac{1}{4}\pi$	$\frac{1}{6}\pi$	0

42a $\arcsin(x)=\frac{1}{2}\pi \Rightarrow x=\sin(\frac{1}{2}\pi)=1$.

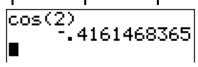
42b $\arccos(x)=\frac{1}{2}\pi \Rightarrow x=\cos(\frac{1}{2}\pi)=0$.

42c $\arcsin(x)=-\frac{1}{6}\pi \Rightarrow x=\sin(-\frac{1}{6}\pi)=-\frac{1}{2}$.

42d $\arccos(x)=-\frac{1}{6}\pi \Rightarrow$ geen oplossing, want $-\frac{1}{6}\pi<0$.

42e $\arcsin(x)=2 \Rightarrow$ geen oplossing, want $2>\frac{1}{2}\pi$.

42f $\arccos(x)=2 \Rightarrow x=\cos(2)\approx -0{,}416$.



42g $3\arcsin(x-\sqrt{3})=\pi \Rightarrow \arcsin(x-\sqrt{3})=\frac{1}{3}\pi \Rightarrow$

$x-\sqrt{3}=\sin(\frac{1}{3}\pi) \Rightarrow x=\sqrt{3}+\frac{1}{2}\sqrt{3}=1\frac{1}{2}\sqrt{3}$.

42h $3\arccos(x-\sqrt{3})=\pi \Rightarrow \arccos(x-\sqrt{3})=\frac{1}{3}\pi \Rightarrow$

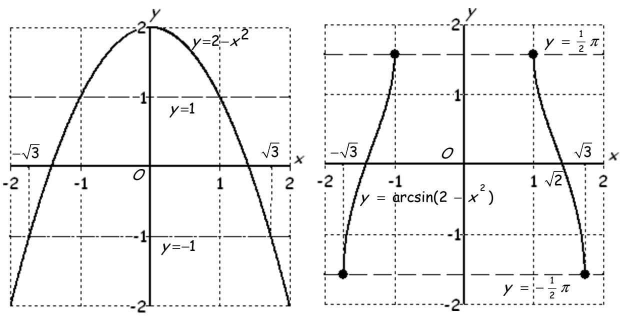
$x-\sqrt{3}=\cos(\frac{1}{3}\pi) \Rightarrow x=\sqrt{3}+\frac{1}{2}$.

43a $f(x)=\arcsin(2-x^2)$ is gedefinieerd voor $-1\le 2-x^2\le 1$.

$2-x^2=-1 \Rightarrow -x^2=-3 \Rightarrow x^2=3 \Rightarrow x=-\sqrt{3} \vee x=\sqrt{3}$

$2-x^2=1 \Rightarrow -x^2=-1 \Rightarrow x^2=1 \Rightarrow x=-1 \vee x=1$.

$-1\le 2-x^2\le 1$ (zie een plot) $\Rightarrow -\sqrt{3}\le x\le -1 \vee 1\le x\le\sqrt{3}$.



43b $B_f=[-\frac{1}{2}\pi,\ \frac{1}{2}\pi]$

43c Maak een schets van de rechter grafiek hiernaast.

43d $f(x)=0 \Rightarrow \arcsin(2-x^2)=0 \Rightarrow 2-x^2=\sin(0) \Rightarrow$

$2-x^2=0 \Rightarrow -x^2=-2 \Rightarrow x^2=2 \Rightarrow x=-\sqrt{2} \vee x=\sqrt{2} \Rightarrow A(\sqrt{2},\,0)$ (zie een plot).

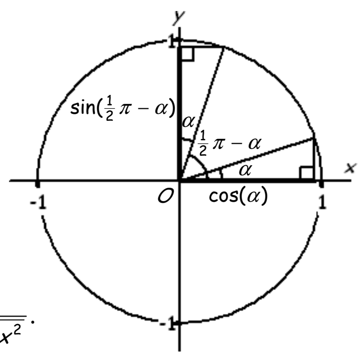
$f(x)=\arcsin(\boxed{2-x^2}) \Rightarrow f'(x)=\frac{1}{\sqrt{1-(2-x^2)^2}}\cdot -2x=\frac{-2x}{\sqrt{1-(2-x^2)^2}}$.

$f'(\sqrt{2})=\frac{-2\cdot\sqrt{2}}{\sqrt{1-(2-2)^2}}=\frac{-2\sqrt{2}}{\sqrt{1}}=-2\sqrt{2} \Rightarrow k$: $y=-2\sqrt{2}x+b$ door $A(\sqrt{2},\,0) \Rightarrow 0=-2\sqrt{2}\cdot\sqrt{2}+b \Rightarrow b=2\sqrt{2}\cdot\sqrt{2}=2\cdot 2=4$.

43e $f(x)=\frac{1}{6}\pi \Rightarrow \arcsin(2-x^2)=\frac{1}{6}\pi \Rightarrow 2-x^2=\sin(\frac{1}{6}\pi) \Rightarrow 2-x^2=\frac{1}{2} \Rightarrow -x^2=-1\frac{1}{2} \Rightarrow x^2=1\frac{1}{2} \Rightarrow$

$x=-\sqrt{1\frac{1}{2}}=-\sqrt{\frac{3}{2}}=-\frac{\sqrt{3}}{\sqrt{2}}=-\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{6}}{2}=-\frac{1}{2}\sqrt{6} \vee x=\sqrt{1\frac{1}{2}}=\frac{1}{2}\sqrt{6}$.

$f(x)<\frac{1}{6}\pi$ (zie een plot) $\Rightarrow -\sqrt{3}\le x<-\frac{1}{2}\sqrt{6} \vee \frac{1}{2}\sqrt{6}<x\le\sqrt{3}$.



44a Uit de symmetrie (zie de eenheidscirkel hiernaast) volgt: $\cos(\alpha)=\sin(\frac{1}{2}\pi-\alpha)$.

44b Stel $\cos(\alpha)=x$, dus ook $\sin(\frac{1}{2}\pi-\alpha)=x$.

$\left.\begin{array}{l}\cos(\alpha)=x \Rightarrow \arccos(x)=\alpha \\ \sin(\frac{1}{2}\pi-\alpha)=x \Rightarrow \arcsin(x)=\frac{1}{2}\pi-\alpha\end{array}\right\} \Rightarrow \arccos(x)+\arcsin(x)=\alpha+\frac{1}{2}\pi-\alpha=\frac{1}{2}\pi$.

44c $[\arcsin(x)+\arccos(x)]'=[\frac{1}{2}\pi]' \Rightarrow \frac{1}{\sqrt{1-x^2}}+[\arccos(x)]'=0 \Rightarrow [\arccos(x)]'=-\frac{1}{\sqrt{1-x^2}}$.

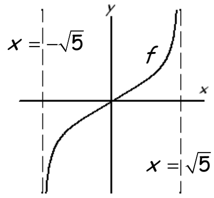
45a $\displaystyle\int_{\frac16}^{\frac16\sqrt3} \frac{1}{\sqrt{1-9x^2}}\,dx = \int_{\frac16}^{\frac16\sqrt3}\frac{1}{\sqrt{1-(3x)^2}}\,dx = \left[\tfrac13\cdot\arcsin(3x)\right]_{\frac16}^{\frac16\sqrt3} = \tfrac13\cdot\arcsin(\tfrac12\sqrt3)-\tfrac13\cdot\arcsin(\tfrac12) = \tfrac13\cdot\tfrac13\pi - \tfrac13\cdot\tfrac16\pi = \tfrac1{18}\pi.$

45b $\displaystyle\int_{-1\frac12\sqrt2}^{1\frac12\sqrt2}\frac{1}{\sqrt{9-x^2}}\,dx = \int_{-1\frac12\sqrt2}^{1\frac12\sqrt2} \frac{\frac13}{\sqrt{1-\frac{x^2}{9}}}\,dx = \int_{-1\frac12\sqrt2}^{1\frac12\sqrt2} \frac{1}{\sqrt{1-(\frac13x)^2}}\,d\left(\tfrac13x\right) = \left[\arcsin(\tfrac13x)\right]_{-1\frac12\sqrt2}^{1\frac12\sqrt2}$

$= \arcsin(\tfrac12\sqrt2) - \arcsin(-\tfrac12\sqrt2) = \tfrac14\pi - -\tfrac14\pi = \tfrac12\pi.$

45c $\displaystyle\int_0^{\frac12\sqrt2}\frac{x}{\sqrt{1-x^4}}\,dx = \int_0^{\frac12\sqrt2}\frac12\cdot\frac{1}{\sqrt{1-(x^2)^2}}\,d\left(x^2\right) = \left[\tfrac12\arcsin(x^2)\right]_0^{\frac12\sqrt2} = \tfrac12\arcsin(\tfrac14\cdot2)-\tfrac12\arcsin(0) = \tfrac12\cdot\tfrac16\pi - 0 = \tfrac1{12}\pi.$

45d $\displaystyle\int_0^1\frac{x}{\sqrt{4-x^4}}\,dx = \int_0^1\frac{\frac12x}{\sqrt{1-\frac14x^4}}\,dx = \int_0^1 \frac{\frac12}{\sqrt{1-(\frac12x^2)^2}}\,d\left(\tfrac12x^2\right) = \left[\tfrac12\arcsin(\tfrac12x^2)\right]_0^1 = \tfrac12\arcsin(\tfrac12\cdot1)-\tfrac12\arcsin(0) = \tfrac12\cdot\tfrac16\pi - 0 = \tfrac1{12}\pi.$



46a Onder een wortelteken mag geen negatief getal staan en een noemer mag niet nul zijn.
Dus $25-x^4>0 \Rightarrow -x^4>-25 \Rightarrow x^4=(x^2)^2<25 \Rightarrow x^2<5 \Rightarrow |x|<\sqrt5 \Rightarrow -\sqrt5<x<\sqrt5.$
Dus $D_f = \langle -\sqrt5, \sqrt5\rangle$. Maak een schets van de grafiek hiernaast.

46b $f(x)=\dfrac{x}{\sqrt{25-x^4}} \Rightarrow f'(x) = \dfrac{\sqrt{25-x^4}\cdot1 - x\cdot\frac{1}{2\cdot\sqrt{25-x^4}}\cdot -4x^3}{\left(\sqrt{25-x^4}\right)^2} = \dfrac{\sqrt{25-x^4}+\frac{2x^4}{\sqrt{25-x^4}}}{25-x^4} = \dfrac{25-x^4+2x^4}{(25-x^4)\sqrt{25-x^4}} = \dfrac{25+x^4}{(25-x^4)\sqrt{25-x^4}}.$

$y_A = f(\sqrt3) = \dfrac{\sqrt3}{\sqrt{25-3^2}} = \dfrac{\sqrt3}{\sqrt{16}} = \dfrac{\sqrt3}{4} = \tfrac14\sqrt3$ en $a = rc_{raaklijn} = f'(\sqrt3) = \dfrac{25+3^2}{(25-3^2)\sqrt{25-3^2}} = \dfrac{34}{16\cdot4} = \dfrac{17}{32}.$

k: $y=\tfrac{17}{32}x+b$ door $A(\sqrt3,\ \tfrac14\sqrt3) \Rightarrow \tfrac14\sqrt3 = \tfrac{17}{32}\cdot\sqrt3+b \Rightarrow \tfrac14\sqrt3-\tfrac{17}{32}\sqrt3 = b \Rightarrow b = \tfrac{8}{32}\sqrt3-\tfrac{17}{32}\sqrt3 = -\tfrac{9}{32}\sqrt3.$

46c $f(p)=\dfrac{p}{\sqrt{25-p^4}}$ en $f(-p)=\dfrac{-p}{\sqrt{25-(-p)^4}} = -\dfrac{p}{\sqrt{25-p^4}} \Rightarrow f(p)=-f(-p)$ voor elke $p \Rightarrow f$ puntsymmetrisch in O.

46d $\displaystyle O(V) = \int_0^{\frac12\sqrt{10}}\frac{x}{\sqrt{25-x^4}}\,dx = \int_0^{\frac12\sqrt{10}} \frac{\frac15x}{\sqrt{1-(\frac15x^2)^2}}\,dx = \int_0^{\frac12\sqrt{10}} \frac12\cdot\frac{\frac25x}{\sqrt{1-(\frac15x^2)^2}}\,dx = \int_0^{\frac12\sqrt{10}}\frac12\cdot\frac{1}{\sqrt{1-(\frac15x^2)^2}}\,d\tfrac15x^2$

$= \left[\tfrac12\arcsin(\tfrac15x^2)\right]_0^{\frac12\sqrt{10}} = \tfrac12\arcsin(\tfrac15\cdot\tfrac14\cdot10) - \arcsin(0) = \tfrac12\arcsin(\tfrac12) = \tfrac12\cdot\tfrac16\pi = \tfrac1{12}\pi.$

47a $\arcsin(x)\,dx = dx\arcsin(x) - x\,d\arcsin(x) = dx\arcsin(x) - x\cdot\dfrac{1}{\sqrt{1-x^2}}dx = dx\arcsin(x)+\dfrac{1}{\sqrt{1-x^2}}\cdot\tfrac12 d(-x^2)$

$= dx\arcsin(x) + \dfrac{1}{2\sqrt{1-x^2}}d(1-x^2) = dx\arcsin(x) + d\sqrt{1-x^2} = d\left(x\arcsin(x)+\sqrt{1-x^2}\right).$

Dus $F(x) = x\arcsin(x)+\sqrt{1-x^2}+c.$

47b $\dfrac{\arcsin(x)}{\sqrt{1-x^2}}dx = \arcsin(x)\,d\arcsin(x) = d\tfrac12\left(\arcsin(x)\right)^2.$ Dus $G(x) = x\arcsin(x)+\sqrt{1-x^2}+c.$

48a $f(x)\,dx = \dfrac{1}{x^2+1}dx = d\arcsin(x) \Rightarrow F(x) = \arcsin(x).$

$g(x)\,dx = \dfrac{x}{x^2+1}dx = \dfrac12\cdot\dfrac{2x}{x^2+1}dx = \dfrac12\cdot\dfrac{1}{x^2+1}dx^2 = \dfrac12\cdot\dfrac{1}{x^2+1}d(x^2+1).$ Dus $G(x) = \tfrac12\cdot\ln(x^2+1).$

48b $f(x)+g(x) = \dfrac{1}{x^2+1}+\dfrac{x}{x^2+1} = \dfrac{x+1}{x^2+1} = h(x).$

48c $h(x)=f(x)+g(x) \Rightarrow H(x) = F(x)+G(x) = \arctan(x)+\tfrac12\cdot\ln(x^2+1).$

49a $\dfrac{2x}{x+1}+\dfrac{5}{x+1} = \dfrac{2x+5}{x+1} = f(x).$ We kennen (nog) geen primitieve van $g(x) = \dfrac{2x}{x+1}.$

49b $2+\dfrac{3}{x+1} = 2\cdot\dfrac{x+1}{x+1}+\dfrac{3}{x+1} = \dfrac{2\cdot(x+1)}{x+1}+\dfrac{3}{x+1} = \dfrac{2x+2}{x+1}+\dfrac{3}{x+1} = \dfrac{2x+5}{x+1} = f(x).$

$f(x)\,dx = (2+\dfrac{3}{x+1})dx = d\left(2x+3\cdot\ln|x+1|\right) \Rightarrow F(x) = 2x+3\cdot\ln|x+1|+c.$

50a $f(x) = \dfrac{2x+1}{x+1} = 2-\dfrac{1}{x+1} \Rightarrow F(x) = 2x-\ln|x+1|+c.$

$x+1/2x+1\ \backslash\ 2$
$\underline{2x+2}\ -$
-1

50b $f(x) = \dfrac{x}{x+1} = 1-\dfrac{1}{x+1} \Rightarrow F(x) = x-\ln|x+1|+c.$

$x+1/x\ \backslash\ 1$
$\underline{x+1}\ -$
-1

50c $f(x)=\frac{x+1}{2x+1}=\frac{1}{2}+\frac{\frac{1}{2}}{2x+1}\Rightarrow F(x)=\frac{1}{2}x+\frac{1}{4}\cdot\ln|2x+1|+c.$

$2x+1/x+1\ \backslash\frac{1}{2}$
$x+\frac{1}{2}\ -$
$\frac{1}{2}$

$x+1/-x+2\ \backslash-1$
$-x-1\ -$
3

50d $f(x)=\frac{2-x}{x+1}=\frac{-x+2}{x+1}=-1+\frac{3}{x+1}\Rightarrow F(x)=-x+3\ln|x+1|+c.$

50e $f(x)=\frac{3-4x}{2x+1}=\frac{-4x+3}{2x+1}=-2+\frac{5}{2x+1}\Rightarrow F(x)=-2x+\frac{5}{2}\cdot\ln|2x+1|+c.$

$2x+1/-4x+3\ \backslash-2$
$-4x-2\ -$
5

$2x-1/-6x+1\ \backslash-3$
$-6x+3\ -$
-2

50f $f(x)=\frac{6x-1}{1-2x}=\frac{-6x+1}{2x-1}=-3-\frac{2}{2x-1}\Rightarrow F(x)=-3x-\ln|2x-1|+c.$

51a $\int_2^3\frac{x^2-2x+3}{x-1}\,dx=\int_2^3\left(x-1+\frac{2}{x-1}\right)dx=\left[\frac{1}{2}x^2-x+2\ln|x-1|\right]_2^3=\frac{9}{2}-3+2\ln(2)-\left(\frac{4}{2}-2+\ln(0)\right)=1\frac{1}{2}+2\ln(2).$

$x-1/x^2-2x+3\ \backslash x-1$
$x^2-x\ -$
$-x$
$-x+1\ -$
2

51b $\int_1^2\frac{x^2+7x}{2x+1}\,dx=\int_1^2\left(\frac{1}{2}x+3\frac{1}{4}-\frac{3\frac{1}{4}}{2x+1}\right)dx=\left[\frac{1}{4}x^2+3\frac{1}{4}x-\frac{13}{8}\ln|2x+1|\right]_1^2$

$=1+6\frac{1}{2}-\frac{13}{8}\ln(5)-\left(\frac{1}{4}+3\frac{1}{4}-\frac{13}{8}\ln(3)\right)$

$=7\frac{1}{2}-\frac{13}{8}\ln(5)-3\frac{1}{2}+\frac{13}{8}\ln(3)=4+\frac{13}{8}\ln(\frac{3}{5}).$

$2x+1/x^2+7x\ \backslash\frac{1}{2}x+3\frac{1}{4}$
$x^2+\frac{1}{2}x\ -$
$6\frac{1}{2}x$
$6\frac{1}{2}x+3\frac{1}{4}\ -$
$-3\frac{1}{4}$

51c $\int_{-4}^{-2}\frac{-2x^2-x}{x+1}\,dx=\int_{-4}^{-2}\left(-2x+1-\frac{1}{x+1}\right)dx=\left[-x^2+x-\ln|x+1|\right]_{-4}^{-2}$

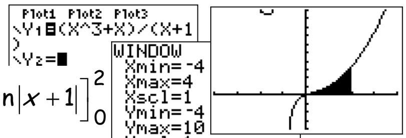
$=-4-2-\ln(1)-(-16-4-\ln(3))=-6+20+\ln(3)=14+\ln(3).$

$x+1/-2x^2-x\ \backslash-2x+1$
$-2x^2-2x\ -$
x
$x+1\ -$
-1

52a $f(x)=\frac{x^3+x}{x+1}\Rightarrow f'(x)=\frac{(x+1)\cdot(3x^2+1)-(x^3+x)\cdot1}{(x+1)^2}=\frac{3x^3+x+3x^2+1-x^3-x}{(x+1)^2}=\frac{2x^3+3x^2+1}{(x+1)^2}.$

$f'(1)=\frac{2+3+1}{(1+1)^2}=\frac{6}{4}=1\frac{1}{2}\Rightarrow k\colon y=1\frac{1}{2}x+b$
$f(1)=\frac{1+1}{1+1}=\frac{2}{2}=1\Rightarrow A(1,1)$
$\Rightarrow 1=1\frac{1}{2}\cdot1+b\Rightarrow-\frac{1}{2}=b.$ Dus $k\colon y=1\frac{1}{2}x-\frac{1}{2}.$

52b $f(x)=0$ (teller = 0) $\Rightarrow x^3+x=0\Rightarrow x\cdot(x^2+1)=0\Rightarrow x=0\ \vee\ x^2=-1\Rightarrow x=0.$



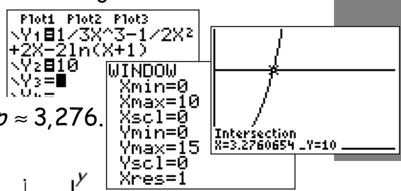
$O(V)=\int_0^2 f(x)dx=\int_0^2\frac{x^3+x}{x+1}\,dx=\int_0^2\left(x^2-x+2-\frac{2}{x+1}\right)dx=\left[\frac{1}{3}x^3-\frac{1}{2}x^2+2x-2\ln|x+1|\right]_0^2$

$=\frac{8}{3}-2+4-2\ln(3)-(0-0+0-2\ln(1))=4\frac{2}{3}-2\ln(3).$

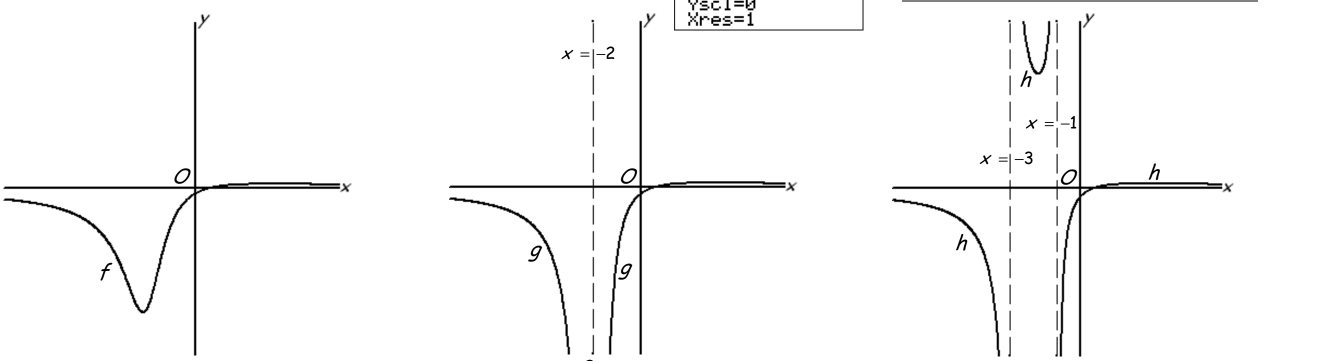
$x+1/x^3\quad+x\ \backslash x^2-x+2$
$x^3+x^2\ -$
$-x^2$
$-x^2-x\ -$
$2x$
$2x+2\ -$
-2

52c $O(W)=\int_0^p f(x)dx=\left[\frac{1}{3}x^3-\frac{1}{2}x^2+2x-2\ln|x+1|\right]_0^p$

$=\frac{1}{3}p^3-\frac{1}{2}p^2+2p-2\ln(p+1)-0=10$ (intersect) $\Rightarrow p\approx3{,}276.$



53



$f(x)$ is voor elke x gedefinieerd, want de noemer x^2+4x+5 heeft geen nulpunten ($D=4^2-4\cdot1\cdot5=16-20<0$).
$g(x)$ is niet voor elke x gedefinieerd, want de noemer $x^2+4x+4=(x+2)^2$ heeft één nulpunt ($\Rightarrow$ V.A.: $x=-2$).
$h(x)$ is niet voor elke x gedefinieerd, want $x^2+4x+3=(x+3)(x+1)$ heeft twee nulpunten ($\Rightarrow$ V.A.: $x=-3$ en $x=-1$).

54a $f(x)=\frac{x^2+x}{x^2+1}=1+\frac{x-1}{x^2+1}=1+\frac{x}{x^2+1}-\frac{1}{x^2+1}\Rightarrow F(x)=x+\frac{1}{2}\cdot\ln(x^2+1)-\arctan(x)+c.$

$x^2+1/x^2+x\ \backslash1$
$x^2\quad+1\ -$
$x-1$

54b $g(x)=1-\frac{1}{x^2-6x+9}=1-\frac{1}{(x-3)^2}=1-(x-3)^{-2}\Rightarrow G(x)=x-\frac{1}{-1}\cdot(x-3)^{-1}+c=x+\frac{1}{x-3}+c.$

$x^2-6x+9/x^2-6x+8\ \backslash1$
$x^2-6x+9\ -$
-1

54c $h(x)=\frac{x^3+x}{x^2+1}=x\Rightarrow H(x)=\frac{1}{2}x^2+c.$

$x^2+1/x^3+x\ \backslash x$
$x^3+x\ -$
0

$x^2-6x+9\,/\,x^3 \qquad +x \quad \backslash\, x+6$
x^3-6x^2+9x −
$6x^2-8x$
$6x^2-36x+54$ −
$28x-54$

54d $j(x)=\frac{x^3+x}{x^2-6x+9}=x+6+\frac{28x-54}{x^2-6x+9}=x+6+\frac{28x-54}{(x-3)^2}$
$=x+6+\frac{28(x-3)+84-54}{(x-3)^2}=x+6+\frac{28}{x-3}+\frac{30}{(x-3)^2}=x+6+\frac{28}{x-3}+30\cdot(x-3)^{-2}.$
$J(x)=\frac{1}{2}x^2+6x+28\cdot\ln|x-3|-30\cdot(x-3)^{-1}+c=\frac{1}{2}x^2+6x+28\ln|x-3|-\frac{30}{x-3}+c.$

55a $\int_0^2 \frac{x^3}{x^2+4}\,dx=\int_0^2\left(x-\frac{4x}{x^2+4}\right)dx=\left[\frac{1}{2}x^2-2\ln(x^2+4)\right]_0^2$
$=\frac{1}{2}\cdot 4-2\ln(8)-(0-2\ln(4))=2-2\ln(8)+2\ln(4)$
$=2-2(\ln(8)-\ln(4))=2-2\ln(\frac{8}{4})=2-2\ln(2).$

$x^2+4\,/\,x^3 \quad \backslash\, x$
x^3+4x −
$-4x$

$x^2+4x+4\,/\,x^3 \quad \backslash\, x-4$
x^3+4x^2+4x −
$-4x^2-4x$
$-4x^2-16x-16$ −
$12x+16$

55b $\frac{x^3}{x^2+4x+4}=x-4+\frac{12x+16}{x^2+4x+4}=x-4+\frac{12\cdot(x+2)-8}{(x+2)^2}=x-4+\frac{12}{x+2}-\frac{8}{(x+2)^2}=x-4+\frac{12}{x+2}-8\cdot(x+2)^{-2}.$
$\int_0^8 \frac{x^3}{x^2+4x+4}\,dx=\left[\frac{1}{2}x^2-4x+12\ln|x+2|+8(x+2)^{-1}\right]_0^8=\left[\frac{1}{2}x^2-4x+12\ln|x+2|+\frac{8}{x+2}\right]_0^8$
$=32-32+12\ln(10)+\frac{8}{10}-\left(0-0+12\ln(2)+\frac{8}{2}\right)=12\ln(10)+\frac{4}{5}-12\ln(2)-4=12\ln(5)-3\frac{1}{5}.$

55c $\int_0^1 \frac{x^4+1}{x^2+1}\,dx=\int_0^1 x^2-1+\frac{2}{x^2+1}\,dx=\left[\frac{1}{3}x^3-x+2\arctan(x)\right]_0^1$
$=\frac{1}{3}-1+2\arctan(1)-(0+2\arctan(0))=-\frac{2}{3}+2\cdot\frac{1}{4}\pi=-\frac{2}{3}+\frac{1}{2}\pi.$

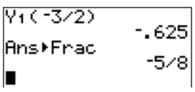
$x^2+1\,/\,x^4 \qquad +1 \quad \backslash\, x^2-1$
x^4+x^2 −
$-x^2$
$-x^2-1$ −
2

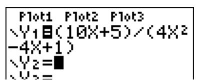
55d $\frac{x^4+1}{x^2+2x+1}=x^2-2x+3-\frac{4x+2}{(x+1)^2}=x^2-2x+3-\frac{4\cdot(x+1)-2}{(x+1)^2}=x^2-2x+3-\frac{4}{x+1}+\frac{2}{(x+1)^2}.$
$\int_0^1 \frac{x^4+1}{x^2+2x+1}\,dx=\left[\frac{1}{3}x^3-x^2+3x-4\ln|x+1|-\frac{2}{x+1}\right]_0^1$
$=\frac{1}{3}-1+3-4\ln(2)-\frac{2}{2}-\left(0-4\ln(1)-\frac{2}{1}\right)=3\frac{1}{3}-4\ln(2).$

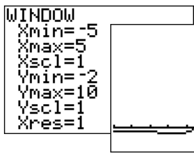
$x^2+2x+1\,/\,x^4 \qquad +1\,\backslash\, x^2-2x+3$
$x^4+2x^3+\ x^2$ −
$-2x^3-\ x^2$
$-2x^3-4x^2-2x$ −
$3x^2+2x$
$3x^2+6x+3$ −
$-4x-2$

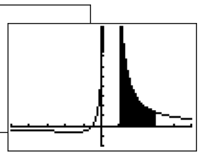
56a $f(x)=\frac{10x+5}{4x^2-4x+1}$ (teller = 0) $\Rightarrow 10x+5=0\Rightarrow 10x=-5\Rightarrow x=-\frac{1}{2}.$
$f'(x)=\frac{(4x^2-4x+1)\cdot 10-(10x+5)\cdot(8x-4)}{(4x^2-4x+1)^2}=\frac{40x^2-40x+10-(80x^2-40x+40x-20)}{(2x-1)^4}=\frac{-40x^2-40x+30}{(2x-1)^4}.$
$f'(-\frac{1}{2})=\frac{-10+20+30}{(-2)^4}=\frac{40}{16}=\frac{5}{2}$
door $A(-\frac{1}{2},\,0)$
$\Big\}\Rightarrow k\colon\ y=\frac{5}{2}x+b$ met $0=\frac{5}{2}\cdot-\frac{1}{2}+b\Rightarrow\frac{5}{4}=b.$ Dus $k\colon\ y=2\frac{1}{2}x+1\frac{1}{4}.$

56b $f'(x)=\frac{-40x^2-40x+30}{(2x-1)^4}=0$ (teller = 0) $\Rightarrow 4x^2+4x-3=0\Rightarrow D=4^2-4\cdot4\cdot-3=16+48=64\Rightarrow x=\frac{-4\pm\sqrt{64}}{2\cdot4}\Rightarrow$
$x=\frac{-4-8}{8}=\frac{-12}{8}=-1\frac{1}{2}$ (voldoet) $\vee\ x=\frac{-4+8}{8}=\frac{4}{8}=\frac{1}{2}$ (vold. niet omdat dan de noemer nul is).
Minimum (zie plot) is $f(-1\frac{1}{2})=\frac{-15+5}{9+6+1}=\frac{-10}{16}=-\frac{5}{8}.$
Dus $B_f=\left[-\frac{5}{8},\ \rightarrow\right\rangle.$









56c $O(V)=\int_1^3 f(x)dx=\int_1^3\frac{10x+5}{(2x-1)^2}\,dx=\int_1^3\frac{5\cdot(2x-1)+10}{(2x-1)^2}\,dx=\int_1^3\left(\frac{5}{2x-1}+10\cdot(2x-1)^{-2}\right)dx$
$=\left[\frac{5}{2}\ln|2x-1|+10\cdot\frac{1}{-1}\cdot\frac{1}{2}\cdot(2x-1)^{-1}\right]_1^3=\left[\frac{5}{2}\ln|2x-1|-\frac{5}{2x-1}\right]_1^3=\frac{5}{2}\ln(5)-\frac{5}{5}-\left(\frac{5}{2}\ln(1)-\frac{5}{1}\right)=2\frac{1}{2}\ln(5)+4.$

57a $\frac{-1\frac{1}{2}}{x+1}+\frac{3\frac{1}{2}}{x+3}=\frac{-1\frac{1}{2}}{x+1}\cdot\frac{x+3}{x+3}+\frac{3\frac{1}{2}}{x+3}\cdot\frac{x+1}{x+1}=\frac{-1\frac{1}{2}\cdot(x+3)}{(x+1)\cdot(x+3)}+\frac{3\frac{1}{2}\cdot(x+1)}{(x+1)\cdot(x+3)}=\frac{-1\frac{1}{2}x-4\frac{1}{2}+3\frac{1}{2}x+3\frac{1}{2}}{x^2+4x+3}=\frac{2x-1}{x^2+4x+3}=h(x).$

57b $h(x)=\frac{2x-1}{x^2+4x+3}$ (zie 57a hierboven) $=\frac{-1\frac{1}{2}}{x+1}+\frac{3\frac{1}{2}}{x+3}\Rightarrow H(x)=-1\frac{1}{2}\ln|x+1|+3\frac{1}{2}\ln|x+3|+c.$

58 $\frac{A}{x-2}+\frac{B}{x-2}=\frac{A+B}{x-2}=\frac{(A+B)\cdot(x-2)}{(x-2)\cdot(x-2)}=\frac{(A+B)\cdot x-2\cdot(A+B)}{(x-2)^2}.$
Wil $\frac{A}{x-2}+\frac{B}{x-2}=\frac{x+1}{(x-2)\cdot(x-2)}$ dan moet $A+B=1$ en $-2(A+B)=1.$
$\begin{cases}A+B=1 & |\times2|\\ -2A-2B=1 & |\times1|\end{cases}\Rightarrow\begin{cases}2A+2B=2\\ -2A-2B=1\end{cases}$ +
$0=3$ (kan niet) $\Rightarrow$ er zijn geen oplossingen.

59a

$$f(x)=\frac{x^2+2x-1}{x^2-5x+6}=1+\frac{7x-7}{x^2-5x+6}=1+\frac{7x-7}{(x-2)\cdot(x-3)}=1+\frac{A}{x-2}+\frac{B}{x-3}=1+\frac{A\cdot(x-3)}{(x-2)\cdot(x-3)}+\frac{B\cdot(x-2)}{(x-2)\cdot(x-3)}$$

$$=1+\frac{A(x-3)+B(x-2)}{(x-2)\cdot(x-3)}=1+\frac{Ax-3A+Bx-2B}{(x-2)\cdot(x-3)}=1+\frac{(A+B)x-3A-2B}{(x-2)\cdot(x-3)}.$$

$$\begin{array}{l} x^2-5x+6\,/\,x^2+2x-1\;\backslash\,1 \\ \quad\underline{x^2-5x+6}\;- \\ \quad 7x-7 \end{array}$$

$$\begin{cases} A+B=7 & |\times 2| \\ -3A-2B=-7 & |\times 1| \end{cases} \Rightarrow \begin{cases} 2A+2B=14 \\ \underline{-3A-2B=-7}\;+ \\ -A\quad=7 \end{cases} \Rightarrow \begin{cases} A=-7 \\ A+B=7 \end{cases} \Rightarrow -7+B=7 \Rightarrow B=14.$$

$$f(x)=1-\frac{7}{x-2}+\frac{14}{x-3}\Rightarrow F(x)=x-7\ln|x-2|+14\ln|x-3|+c.$$

59b

$$g(x)=\frac{x^3}{x^2+2x-3}=x-2+\frac{7x-6}{x^2+2x-3}=x-2+\frac{7x-6}{(x-1)\cdot(x+3)}=x-2+\frac{A}{x-1}+\frac{B}{x+3}=x-2+\frac{A\cdot(x+3)}{(x-1)\cdot(x+3)}+\frac{B\cdot(x-1)}{(x-1)\cdot(x+3)}$$

$$=x-2+\frac{Ax+3A+Bx-B}{(x-1)\cdot(x+3)}=x-2+\frac{(A+B)x+3A-B}{(x-1)\cdot(x+3)}.$$

$$\begin{array}{l} x^2+2x-3\,/\,x^3\qquad\backslash\,x-2 \\ \quad\underline{x^3+2x^2-3x}\;- \\ \quad -2x^2+3x \\ \quad\underline{-2x^2-4x+6}\;- \\ \quad 7x-6 \end{array}$$

$$\begin{cases} A+B=7 \\ \underline{3A-B=-6}\;+ \\ 4A\quad=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{4} \\ A+B=7 \end{cases} \Rightarrow \tfrac{1}{4}+B=7 \Rightarrow B=6\tfrac{3}{4}.$$

$$g(x)=x-2+\frac{\frac{1}{4}}{x-1}+\frac{6\frac{3}{4}}{x+3}\Rightarrow G(x)=\tfrac{1}{2}x^2-2x+\tfrac{1}{4}\ln|x-1|+6\tfrac{3}{4}\ln|x+3|+c.$$

59c

$$h(x)=\frac{2x^2+1}{2x^2-x}=1+\frac{x+1}{2x^2-x}=1+\frac{x+1}{x\cdot(2x-1)}=1+\frac{A}{x}+\frac{B}{2x-1}=1+\frac{A\cdot(2x-1)}{x\cdot(2x-1)}+\frac{B\cdot x}{x\cdot(2x-1)}=1+\frac{2Ax-A+Bx}{x\cdot(2x-1)}=1+\frac{(2A+B)x-A}{x\cdot(2x-1)}.$$

$$\begin{cases} 2A+B=1 \\ -A=1 \end{cases} \Rightarrow -2+B=1 \Rightarrow B=-3.$$

$$\begin{array}{l} 2x^2-x\,/\,2x^2\quad+1\;\backslash\,1 \\ \quad\underline{2x^2-x}\;- \\ \quad x+1 \end{array}$$

$$h(x)=1-\frac{1}{x}-\frac{3}{2x-1}\Rightarrow H(x)=x-\ln|x|-1\tfrac{1}{2}\ln|2x-1|+c.$$

59d

$$j(x)=\frac{2x^3+1}{2x^2+x}=x-\frac{1}{2}+\frac{\frac{1}{2}x+1}{2x^2+x}=x-\frac{1}{2}+\frac{\frac{1}{2}x+1}{x\cdot(2x+1)}=x-\frac{1}{2}+\frac{A}{x}+\frac{B}{2x+1}=x-\frac{1}{2}+\frac{A\cdot(2x+1)}{x\cdot(2x+1)}+\frac{B\cdot x}{x\cdot(2x+1)}$$

$$=x-\frac{1}{2}+\frac{2Ax+A+Bx}{x\cdot(2x+1)}=x-\frac{1}{2}+\frac{(2A+B)x+A}{x\cdot(2x+1)}.$$

$$\begin{array}{l} 2x^2+x\,/\,2x^3\qquad+1\;\backslash\,x-\frac{1}{2} \\ \quad\underline{2x^3+x^2}\;- \\ \quad -x^2 \\ \quad\underline{-x^2-\frac{1}{2}x}\;- \\ \quad \frac{1}{2}x+1 \end{array}$$

$$\begin{cases} 2A+B=\frac{1}{2} \\ A=1 \end{cases} \Rightarrow 2+B=\tfrac{1}{2} \Rightarrow B=-1\tfrac{1}{2}.$$

$$j(x)=x-\frac{1}{2}+\frac{1}{x}-\frac{1\frac{1}{2}}{2x+1}\Rightarrow J(x)=\tfrac{1}{2}x^2-\tfrac{1}{2}x+\ln|x|-\tfrac{3}{4}\ln|2x+1|+c.$$

60a

$$\frac{x}{x^2-x-2}=\frac{x}{(x+1)\cdot(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}=\frac{A\cdot(x-2)}{(x+1)\cdot(x-2)}+\frac{B\cdot(x+1)}{(x+1)\cdot(x-2)}=\frac{Ax-2A+Bx+B}{(x+1)\cdot(x-2)}=\frac{(A+B)x-2A+B}{(x+1)\cdot(x-2)}.$$

$$\begin{cases} A+B=1 \\ \underline{-2A+B=0}\;- \\ 3A\quad=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{3} \\ A+B=1 \end{cases} \Rightarrow \tfrac{1}{3}+B=1 \Rightarrow B=\tfrac{2}{3}.$$

$$\int_3^4 \frac{x}{x^2-x-2}\,dx=\int_3^4\left(\frac{\frac{1}{3}}{x+1}+\frac{\frac{2}{3}}{x-2}\right)dx=\left[\tfrac{1}{3}\ln|x+1|+\tfrac{2}{3}\ln|x-2|\right]_3^4$$

$$=\tfrac{1}{3}\ln(5)+\tfrac{2}{3}\ln(2)-\left(\tfrac{1}{3}\ln(4)+\tfrac{2}{3}\ln(1)\right)=\tfrac{1}{3}\ln(5)+\tfrac{2}{3}\ln(2)-\tfrac{1}{3}\ln(2^2)=\tfrac{1}{3}\ln(5)+\tfrac{2}{3}\ln(2)-\tfrac{2}{3}\ln(2)=\tfrac{1}{3}\ln(5).$$

60b

$$\frac{4x-8}{x^2-4x-5}=\frac{4x-8}{(x+1)\cdot(x-5)}=\frac{A}{x+1}+\frac{B}{x-5}=\frac{A\cdot(x-5)}{(x+1)\cdot(x-5)}+\frac{B\cdot(x+1)}{(x+1)\cdot(x-5)}=\frac{Ax-5A+Bx+B}{(x+1)\cdot(x-5)}=\frac{(A+B)x-5A+B}{(x+1)\cdot(x-5)}.$$

$$\begin{cases} A+B=4 \\ \underline{-5A+B=-8}\;- \\ 6A\quad=12 \end{cases} \Rightarrow \begin{cases} A=2 \\ A+B=4 \end{cases} \Rightarrow 2+B=4 \Rightarrow B=2.$$

$$\int_0^2 \frac{4x-8}{x^2-4x-5}\,dx=\int_0^2\left(\frac{2}{x+1}+\frac{2}{x-5}\right)dx=\left[2\ln|x+1|+2\ln|x-5|\right]_0^2=2\ln(3)+2\ln(3)-\left(2\ln(1)+2\ln(5)\right)=4\ln(3)-2\ln(5).$$

60c

$$\frac{-2}{x^2-2x-3}=\frac{-2}{(x+1)\cdot(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}=\frac{A\cdot(x-3)}{(x+1)\cdot(x-3)}+\frac{B\cdot(x+1)}{(x+1)\cdot(x-3)}=\frac{Ax-3A+Bx+B}{(x+1)\cdot(x-3)}=\frac{(A+B)x-3A+B}{(x+1)\cdot(x-3)}.$$

$$\begin{cases} A+B=0 \\ \underline{-3A+B=-2}\;- \\ 4A\quad=2 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{2} \\ A+B=0 \end{cases} \Rightarrow \tfrac{1}{2}+B=0 \Rightarrow B=-\tfrac{1}{2}.$$

$$\int_0^2 \frac{-2}{x^2-2x-3}\,dx=\int_0^2\left(\frac{\frac{1}{2}}{x+1}-\frac{\frac{1}{2}}{x-3}\right)dx=\left[\tfrac{1}{2}\ln|x+1|-\tfrac{1}{2}\ln|x-3|\right]_0^2$$

$$=\tfrac{1}{2}\ln(3)-\tfrac{1}{2}\ln(1)-\left(\tfrac{1}{2}\ln(1)-\tfrac{1}{2}\ln(3)\right)=\ln(3).$$

$$\begin{array}{l} x^2+4x+3\,/\,x^4\qquad\backslash\,x^2-4x+13 \\ \quad\underline{x^4+4x^3+3x^2}\;- \\ \quad -4x^3-3x^2 \\ \quad\underline{-4x^3-16x^2-12x}\;- \\ \quad 13x^2+12x \\ \quad\underline{13x^2+52x+39}\;- \\ \quad -40x-39 \end{array}$$

60d

$$\frac{x^4}{x^2+4x+3}=x^2-4x+13+\frac{-40x-39}{(x+3)\cdot(x+1)}.$$

$$\frac{-40x-39}{(x+3)\cdot(x+1)}=\frac{A}{x+3}+\frac{B}{x+1}=\frac{A\cdot(x+1)}{(x+3)\cdot(x+1)}+\frac{B\cdot(x+3)}{(x+3)\cdot(x+1)}=\frac{Ax+A+Bx+3B}{(x+3)\cdot(x+1)}=\frac{(A+B)x+A+3B}{(x+3)\cdot(x+1)}.$$

$$\begin{cases} A+B=-40 \\ \underline{A+3B=-39}\;- \\ -2B=-1 \end{cases} \Rightarrow \begin{cases} B=\frac{1}{2} \\ A+B=-40 \end{cases} \Rightarrow A+\tfrac{1}{2}=-40 \Rightarrow A=-40\tfrac{1}{2}.$$

$$\int_0^2 \frac{x^4}{x^2+4x+3}\,dx=\int_0^2\left(x^2-4x+13-\frac{40\frac{1}{2}}{x+3}+\frac{\frac{1}{2}}{x+1}\right)dx=\left[\tfrac{1}{3}x^3-2x^2+13x-40\tfrac{1}{2}\ln|x+3|+\tfrac{1}{2}\ln|x+1|\right]_0^2$$

$$=\tfrac{1}{3}\cdot 2^3-2\cdot 2^2+13\cdot 2-40\tfrac{1}{2}\ln(5)+\tfrac{1}{2}\ln(3)-\left(0-0+0-40\tfrac{1}{2}\ln(3)+\tfrac{1}{2}\ln(1)\right)$$

$$=2\tfrac{2}{3}-8+26-40\tfrac{1}{2}\ln(5)+\tfrac{1}{2}\ln(3)+40\tfrac{1}{2}\ln(3)=20\tfrac{2}{3}+41\ln(3)-40\tfrac{1}{2}\ln(5).$$

61 I $f(x)=\frac{2x-5}{x^2-5x+6}=\frac{2x-5}{(x-2)\cdot(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}=\frac{A\cdot(x-3)}{(x-2)\cdot(x-3)}+\frac{B\cdot(x-2)}{(x-2)\cdot(x-3)}=\frac{Ax-3A+Bx-2B}{(x-2)\cdot(x-3)}=\frac{(A+B)x-3A-2B}{(x-2)\cdot(x-3)}.$

$$\begin{cases}A+B=2 & |\times 2\\ -3A-2B=-5 & |\times 1\end{cases}\Rightarrow\begin{cases}2A+2B=4\\ \underline{-3A-2B=-5}\ +\\ -A=-1\end{cases}\Rightarrow\begin{cases}A=1\\ A+B=2\end{cases}\Rightarrow 1+B=2\Rightarrow B=1.$$

$f(x)=\frac{1}{x-2}+\frac{1}{x-3}\Rightarrow F(x)=\ln|x-2|+\ln|x-3|+c.$

II $f(x)\,dx=\frac{2x-5}{x^2-5x+6}\,dx=\frac{1}{x^2-5x+6}\,d\left(x^2-5x+6\right)=d\ln\left|x^2-5x+6\right|\Rightarrow F(x)=\ln\left|x^2-5x+6\right|+c.$

Nu is: $\ln|x-2|+\ln|x-3|=\ln|(x-2)\cdot(x-3)|=\ln\left|x^2-5x+6\right|\Rightarrow$ de primitieven komen op hetzelfde neer.

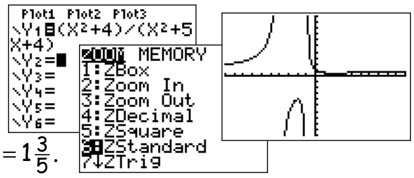
62a Noemer $=0\Rightarrow x^2+5x+4=0\Rightarrow(x+4)\cdot(x+1)=0\Rightarrow x=-4\vee x=-1.$ Dus verticale asymptoten: $x=-4$ en $x=-1$.

$f(x)=\frac{x^2+4}{x^2+5x+4}\Rightarrow f'(x)=\frac{(x^2+5x+4)\cdot 2x-(x^2+4)\cdot(2x+5)}{(x^2+5x+4)^2}=\frac{2x^3+10x^2+8x-(2x^3+5x^2+8x+20)}{(x^2+5x+4)^2}=\frac{5x^2-20}{(x^2+5x+4)^2}.$

$f'(x)=0$ (teller $=0$) $\Rightarrow 5x^2-20=0\Rightarrow 5x^2=20\Rightarrow x^2=4\Rightarrow x=-2\vee x=2.$

Maximum (zie plot): $f(-2)=\frac{4+4}{4-10+4}=\frac{8}{-2}=-4.$

Minimum (zie plot): $f(2)=\frac{4+4}{4+10+4}=\frac{8}{18}=\frac{4}{9}.$



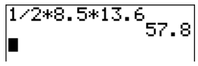
62b $f(-6)=\frac{36+4}{36-30+4}=\frac{40}{10}=4\Rightarrow A(-6,4)$ en $f'(-6)=\frac{5\cdot 36-20}{(36-30+4)^2}=\frac{160}{100}=\frac{16}{10}=\frac{8}{5}=1\frac{3}{5}.$

k: $y=\frac{8}{5}x+b$ door $A(-6,4)\Rightarrow 4=\frac{8}{5}\cdot -6+b\Rightarrow 4+\frac{48}{5}=4+9\frac{3}{5}=13\frac{3}{5}=b.$ Dus k: $y=1\frac{3}{5}x+13\frac{3}{5}.$

k snijden met de x-as ($y=0$) $\Rightarrow 1\frac{3}{5}x+13\frac{3}{5}=0\Rightarrow\frac{8}{5}x=13\frac{3}{5}=\frac{68}{5}\Rightarrow x=\frac{68}{5}\cdot\frac{5}{8}=\frac{68}{8}=8\frac{1}{2}\Rightarrow B(8\frac{1}{2},0).$

k snijden met de y-as ($x=0$) $\Rightarrow y=0+13\frac{3}{5}\Rightarrow C(0,13\frac{3}{5}).$

$O(\Delta ABC)=\frac{1}{2}\cdot OB\cdot OC=\frac{1}{2}\cdot 8\frac{1}{2}\cdot 13\frac{3}{5}=57\frac{4}{5}.$



62c $\frac{x^2+4}{x^2+5x+4}=1+\frac{-5x}{(x+4)(x+1)}=1+\frac{A}{x+4}+\frac{B}{x+1}=1+\frac{A(x+1)}{(x+4)(x+1)}+\frac{B(x+4)}{(x+4)(x+1)}=1+\frac{Ax+A+Bx+4B}{(x+4)(x+1)}=1+\frac{(A+B)x+A+4B}{(x+4)(x+1)}.$

$$\begin{cases}A+B=-5\\ \underline{A+4B=0}\ -\\ -3B=-5\end{cases}\Rightarrow\begin{cases}B=\frac{5}{3}=1\frac{2}{3}\\ A+B=-5\end{cases}\Rightarrow A+1\frac{2}{3}=-5\Rightarrow A=-6\frac{2}{3}.$$

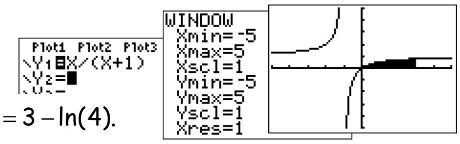
$x^2+5x+4\,/\,x^2\quad+4\quad\backslash\,1$
$\underline{x^2+5x+4}\ -$
$-5x$

$$\int_0^6\frac{x^2+4}{x^2+5x+4}\,dx=\int_0^6\left(1-\frac{6\frac{2}{3}}{x+4}+\frac{1\frac{2}{3}}{x+1}\right)dx=\left[x-6\tfrac{2}{3}\ln|x+4|+1\tfrac{2}{3}\ln|x+1|\right]_0^6$$

$$=6-6\tfrac{2}{3}\ln(10)+1\tfrac{2}{3}\ln(7)-\left(0-6\tfrac{2}{3}\ln(4)+1\tfrac{2}{3}\ln(1)\right)=\ln(3)=6-6\tfrac{2}{3}\ln(10)+1\tfrac{2}{3}\ln(7)+6\tfrac{2}{3}\ln(4)=6+1\tfrac{2}{3}\ln(7)+6\tfrac{2}{3}\ln(\tfrac{2}{5}).$$

63a $f(x)=0$ (teller $=0$) $\Rightarrow x=0.$ $f(x)=\frac{x}{x+1}=1+\frac{-1}{x+1}.$

$x+1\,/\,x\quad\backslash\,1$
$\underline{x+1}\ -$
-1



$O(V)=\int_0^3\frac{x}{x+1}\,dx=\int_0^3\left(1-\frac{1}{x+1}\right)dx=\left[x-\ln|x+1|\right]_0^3=3-\ln(4)-(0-\ln(1))=3-\ln(4).$

63b $\left(\frac{x}{x+1}\right)^2=\frac{x^2}{x^2+2x+1}=1-\frac{2x+1}{x^2+2x+1}=1-\frac{2x+1}{(x+1)^2}=1-\frac{2(x+1)-1}{(x+1)^2}=1-\frac{2}{x+1}+\frac{1}{(x+1)^2}.$

$x^2+2x+1\,/\,x^2\quad\backslash\,1$
$\underline{x^2+2x+1}\ -$
$-2x-1$

$$I(L)=\int_0^3\pi\cdot\left(\frac{x}{x+1}\right)^2dx=\int_0^3\pi\cdot\left(1-\frac{2}{x+1}+(x+1)^{-2}\right)dx=\left[\pi\cdot\left(x-2\ln|x+1|-(x+1)^{-1}\right)\right]_0^3$$

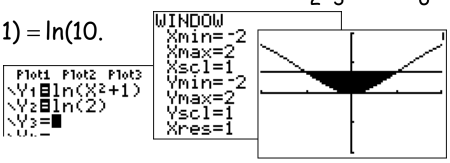
$$=\left[\pi\cdot\left(x-2\ln|x+1|-\frac{1}{x+1}\right)\right]_0^3=\pi\cdot\left(3-2\ln(4)-\tfrac{1}{4}\right)-\pi\cdot(0-2\ln(1)-1)=2\tfrac{3}{4}\pi-2\pi\ln(4)+\pi=3\tfrac{3}{4}\pi-2\pi\ln(4).$$

64a $f(x)=\ln(x^2+1)\Rightarrow f'(x)=\frac{1}{x^2+1}\cdot 2x=\frac{2x}{x^2+1}.$

$f'(x)=\frac{3}{5}\Rightarrow\frac{2x}{x^2+1}=\frac{3}{5}\Rightarrow 3x^2+3=10x\Rightarrow 3x^2-10x+3=0$ ($D=(-10)^2-4\cdot 3\cdot 3=100-36=64$) $\Rightarrow x=\frac{10\pm\sqrt{64}}{2\cdot 3}=\frac{10\pm 8}{6}.$

$x=\frac{10-8}{6}=\frac{2}{6}=\frac{1}{3}\Rightarrow y=\ln(\frac{1}{9}+1)=\ln(\frac{10}{9})$ en $x=\frac{10+8}{6}=\frac{18}{6}=3\Rightarrow y=\ln(9+1)=\ln(10.$

64b $f(x)=\ln(2)\Rightarrow\ln(x^2+1)=\ln(2)\Rightarrow x^2+1=2\Rightarrow x^2=1\Rightarrow x=-1\vee x=1.$



$\ln(x^2+1)\,\mathrm{d}x = \mathrm{d}\left(x\ln(x^2+1)\right) - x\,\mathrm{d}\ln(x^2+1) = \mathrm{d}\left(x\ln(x^2+1)\right) - x\cdot\frac{1}{x^2+1}\cdot 2x\,($ $x^2+1/2x^2 \backslash 2$ $2x^2+2$ − -2

$= \mathrm{d}\left(x\ln(x^2+1)\right) - \frac{2x^2}{x^2+1}\,\mathrm{d}x = \mathrm{d}\left(x\ln(x^2+1)\right) - \left(2-\frac{2}{x^2+1}\right)\mathrm{d}x$

$= \mathrm{d}\left(x\ln(x^2+1)\right) - 2\mathrm{d}x + \frac{2}{x^2+1}\,\mathrm{d}x = \mathrm{d}\left(x\ln(x^2+1)\right) - \mathrm{d}(2x) + \mathrm{d}\,(2\arctan(x)) = \mathrm{d}\left(x\ln(x^2+1) - 2x + 2\arctan(x)\right).$

$O(V) = \int_{-1}^{1}\left(\ln(2)-\ln(x^2+1)\right)\mathrm{d}x = \left[x\ln(2) - x\ln(x^2+1) + 2x - 2\arctan(x)\right]_{-1}^{1}$

$= \ln(2)-\ln(2)+2-2\arctan(1) - (-\ln(2)+\ln(2)-2-2\arctan(-1)) = 2-2\cdot\frac{1}{4}\pi+2+2\cdot-\frac{1}{4}\pi = 2-\frac{1}{2}\pi+2-\frac{1}{2}\pi = 4-\pi.$

Diagnostische toets

D1a $3x^2\cdot(\boxed{x^3+2})^4\,\mathrm{d}x = (\boxed{x^3+2})^4\,\mathrm{d}\boxed{x^3+2} = \mathrm{d}\left(\frac{1}{5}(x^3+2)^5\right) \Rightarrow F(x) = \frac{1}{5}(x^3+2)^5 + c.$

D1b $3x\cdot\sqrt{\boxed{x^2+2}}\,\mathrm{d}x = 3x\cdot(\boxed{x^2+2})^{\frac{1}{2}}\,\mathrm{d}x = \frac{3}{2}\cdot(\boxed{x^2+2})^{\frac{1}{2}}\,\mathrm{d}\boxed{x^2+2} = \mathrm{d}\left(\frac{3}{2}\cdot\frac{1}{1\frac{1}{2}}(x^2+2)^{1\frac{1}{2}}\right) \Rightarrow F(x) = (x^2+2)\cdot\sqrt{x^2+2}+c.$

D1c $\cos(x)\cdot\left(1+(\boxed{\sin(x)})^3\right)\mathrm{d}x = \left(1+(\boxed{\sin(x)})^3\right)\mathrm{d}\boxed{\sin(x)} = \mathrm{d}\left(\sin(x)+\frac{1}{4}\cdot(\sin(x))^4\right) \Rightarrow F(x) = \sin(x)+\frac{1}{4}\cdot\sin^4(x)+c.$

D1d $3x^2\cdot\cos(\boxed{x^3+2})\,\mathrm{d}x = \cos(\boxed{x^3+2})\,\mathrm{d}\boxed{x^3+2} = \mathrm{d}\left(\sin(x^3+2)\right) \Rightarrow F(x) = \sin(x^3+2)+c.$

D1e $\frac{6}{(\boxed{2x-1})^3}\,\mathrm{d}x = 6\cdot(\boxed{2x-1})^{-3}\,\mathrm{d}x = 3\cdot(\boxed{2x-1})^{-3}\,\mathrm{d}\boxed{2x-1} = \mathrm{d}\left(\frac{3}{-2}(2x-1)^{-2}\right) \Rightarrow F(x) = -\frac{3}{2(2x-1)^2}+c.$

D1f $2\cdot(2x+3)\ln(\boxed{x^2+3x})\,\mathrm{d}x = 2\ln(\boxed{x^2+3x})\,\mathrm{d}\boxed{x^2+3x} = \mathrm{d}\left(2\left((x^2+3x)\cdot\ln(x^2+3x)-(x^2+3x)\right)\right).$

Dus $F(x) = 2(x^2+3x)\cdot\ln(x^2+3x) - 2(x^2+3x)+c.$

D2a $\frac{\sqrt{\boxed{\ln(x)}}}{x}\,\mathrm{d}x = (\boxed{\ln(x)})^{\frac{1}{2}}\cdot\frac{1}{x}\,\mathrm{d}x = (\boxed{\ln(x)})^{\frac{1}{2}}\,\mathrm{d}\boxed{\ln(x)} = \mathrm{d}\left(\frac{1}{1\frac{1}{2}}(\ln(x))^{1\frac{1}{2}}\right) = \mathrm{d}\left(\frac{2}{3}\ln(x)\cdot\sqrt{\ln(x)}\right).$

$\int_1^e \frac{\sqrt{\ln(x)}}{x}\,\mathrm{d}x = \left[\frac{2}{3}\ln(x)\cdot\sqrt{\ln(x)}\right]_1^e = \frac{2}{3}\ln(e^1)\cdot\sqrt{\ln(e^1)} - \frac{2}{3}\ln(1)\cdot\sqrt{\ln(1)} = \frac{2}{3}\cdot1\cdot\sqrt{1}-\frac{2}{3}\cdot0\cdot\sqrt{0} = \frac{2}{3}.$

D2b $2x\cdot e^{\boxed{x^2+1}}\,\mathrm{d}x = e^{\boxed{x^2+1}}\,\mathrm{d}\boxed{x^2+1} = \mathrm{d}\left(e^{x^2+1}\right) \Rightarrow \int_0^2 2x\cdot e^{x^2+1}\,\mathrm{d}x = \left[e^{x^2+1}\right]_0^2 = e^{4+1}-e^{0+1} = e^5-e.$

D2c $\sin(x)\cdot\cos^3(x)\,\mathrm{d}x = -(\boxed{\cos(x)})^3\,\mathrm{d}\boxed{\cos(x)} = \mathrm{d}\left(-\frac{1}{4}(\cos(x))^4\right).$

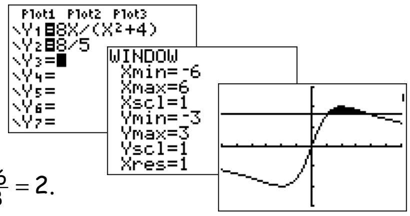
$\int_0^{\frac{1}{2}\pi}\sin(x)\cdot\cos^3(x)\,\mathrm{d}x = \left[-\frac{1}{4}\cos^4(x)\right]_0^{\frac{1}{2}\pi} = -\frac{1}{4}\cos^4(\frac{1}{2}\pi) - -\frac{1}{4}\cos^4(0) = -\frac{1}{4}\cdot0^4+\frac{1}{4}\cdot1^4 = \frac{1}{4}.$

D2d $\frac{6x}{\boxed{x^2+1}}\,\mathrm{d}x = \frac{1}{\boxed{x^2+1}}\cdot6x\,\mathrm{d}x = \frac{1}{\boxed{x^2+1}}\cdot3\,\mathrm{d}\boxed{x^2+1} = \mathrm{d}\left(3\ln(x^2+1)\right).$

$\int_1^3\frac{6x}{x^2+1}\,\mathrm{d}x = \left[3\ln(x^2+1)\right]_1^3 = 3\ln(3^2+1)-3\ln(1^2+1) = 3\ln(10)-3\ln(2) = 3\ln(\frac{10}{2}) = 3\ln(5).$

D3a Beginvoorwaarde: $x^3+4 \neq 0 \Rightarrow x^2 \neq -4$ (zit goed).

$f(x)=\frac{8x}{x^2+4} \Rightarrow f'(x)=\frac{(x^2+4)\cdot 8-8x\cdot 2x}{(x^2+4)^2}=\frac{8x^2+32-16x^2}{(x^2+4)^2}=\frac{-8x^2+32}{(x^2+4)^2}$.



$f'(x)=0 \Rightarrow \frac{-8x^2+32}{(x^2+4)^2}=0$ (teller = 0) $\Rightarrow -8x^2+32=0 \Rightarrow$

$-8x^2=-32 \Rightarrow x^2=4 \Rightarrow x=-2 \vee x=2$.

minimium (zie plot) is $f(-2)=\frac{-16}{8}=-2$ en maximium (zie plot) is $f(2)=\frac{16}{8}=2$.

D3b $f(x)=1\frac{3}{5} \Rightarrow \frac{8x}{x^2+4}=\frac{8}{5} \Rightarrow 8\cdot(x^2+4)=5\cdot 8x \Rightarrow x^2+4=5x \Rightarrow x^2-5x+4=0 \Rightarrow (x-1)(x-4)=0 \Rightarrow x=1 \vee x=4$.

$\frac{8x}{\boxed{x^2+4}}\,dx=\frac{1}{\boxed{x^2+4}}\cdot 8x\,dx=\frac{1}{\boxed{x^2+4}}\cdot 4\,d\boxed{x^2+4}=d\left(4\ln\left|x^2+4\right|\right)$.

$\int_1^4\left(f(x)-1\frac{3}{5}\right)dx=\left[4\ln(x^2+4)-\frac{8}{5}x\right]_1^4=4\ln(20)-\frac{32}{5}-\left(4\ln(5)-\frac{8}{5}\right)=4\ln(\frac{20}{5})-\frac{24}{5}=4\ln(4)-4\frac{4}{5}$.

D3c $f(x)=\frac{8x}{x^2+4}=0$ (teller = 0) $\Rightarrow 8x=0 \Rightarrow x=0 \Rightarrow \int_0^p f(x)\,dx=\left[4\ln(x^2+4)\right]_0^p=4\ln(p^2+4)-4\ln(0^2+4)=4\ln(\frac{p^2+4}{4})$.

$\int_0^p f(x)\,dx=8 \Rightarrow 4\ln(\frac{p^2+4}{4})=8 \Rightarrow \ln(\frac{p^2+4}{4})=2 \Rightarrow \frac{p^2+4}{4}=e^2 \Rightarrow p^2+4=4e^2 \Rightarrow p^2=4e^2-4$ $(p>0) \Rightarrow p=\sqrt{4e^2-4}$.

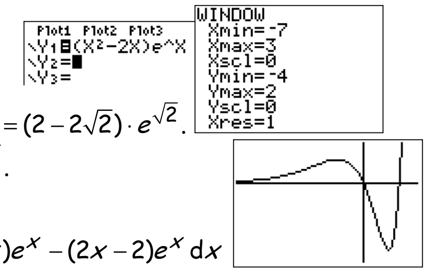
D4a $4x\sin(2x)\,dx=-2x\,d\cos(2x)=d(-2x\cos(2x))+\cos(2x)\,d2x=d(-2x\cos(2x))+2\cos(2x)\,dx$
$=d(-2x\cos(2x))+d\sin(2x)=d(-2x\cos(2x)+\sin(2x))$. Dus $F(x)=-2x\cos(2x)+\sin(2x)+c$.

D4b $(2x+3)e^x\,dx=(2x+3)\,de^x=d(2x+3)e^x-e^x\,d(2x+3)=d(2x+3)e^x-2e^x\,dx$
$=d(2x+3)e^x-d2e^x=d((2x+3)e^x-2e^x)=d((2x+1)e^x)$. Dus $G(x)=(2x+1)e^x+c$.

D4c $x^2\cdot\ln(x)\,dx=\ln(x)\,d\frac{1}{3}x^3=d\left(\frac{1}{3}x^3\cdot\ln(x)\right)-\frac{1}{3}x^3\,d\ln(x)=d\left(\frac{1}{3}x^3\cdot\ln(x)\right)-\frac{1}{3}x^3\cdot\frac{1}{x}\,dx=d\left(\frac{1}{3}x^3\cdot\ln(x)\right)-\frac{1}{3}x^2\,dx$
$=d\left(\frac{1}{3}x^3\cdot\ln(x)\right)-d\frac{1}{9}x^9=d\left(\frac{1}{3}x^3\cdot\ln(x)-\frac{1}{9}x^9\right)$. Dus $H(x)=\frac{1}{3}x^3\cdot\ln(x)-\frac{1}{9}x^9+c$.

D5a $f(x)=(x^2-2x)\cdot e^x \Rightarrow f'(x)=(2x-2)\cdot e^x+(x^2-2x)\cdot e^x=(x^2-2)\cdot e^x$.

$f'(x)=0 \Rightarrow (x^2-2)\cdot e^x=0 \Rightarrow x^2-2=0 \Rightarrow x^2=2 \Rightarrow x=-\sqrt{2} \vee x=\sqrt{2}$.



maximium (zie plot) is $f(-\sqrt{2})=(2+2\sqrt{2})\cdot e^{-\sqrt{2}}$ en minimium (zie plot) is $f(\sqrt{2})=(2-2\sqrt{2})\cdot e^{\sqrt{2}}$.

$f(x)=p$ heeft precies drie oplossingen (zie plot) voor $0<p<(2+2\sqrt{2})\cdot e^{-\sqrt{2}}$.

D5b $f(x)=0 \Rightarrow (x^2-2x)\cdot e^x=0 \Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0 \Rightarrow x=0 \vee x=2$.

$(x^2-2x)e^x\,dx=(x^2-2x)\,de^x=d(x^2-2x)e^x-e^x\,d(x^2-2x)=d(x^2-2x)e^x-(2x-2)e^x\,dx$
$=d(x^2-2x)e^x-(2x-2)\,de^x=d(x^2-2x)e^x-d(2x-2)e^x+e^x\,d(2x-2)$
$=d(x^2-2x)e^x-d(2x-2)e^x+2e^x\,dx=d(x^2-2x)e^x-d(2x-2)e^x+d2e^x$
$=d(x^2-2x-2x+2+2)e^x=d(x^2-4x+4)e^x$.

$O(V)=\int_0^2 -f(x)\,dx=\left[-(x^2-4x+4)e^x\right]_0^2=-(4-8+4)e^2+(0+0+4)e^0=0+4\cdot 1=4$.

D6a $4x^2\sin(2x)\,dx=-2x^2\,d\cos(2x)=d\left(-2x^2\cos(2x)\right)+\cos(2x)\,d2x^2=d\left(-2x^2\cos(2x)\right)+4x\cos(2x)\,dx$
$=d\left(-2x^2\cos(2x)\right)+2x\,d\sin(2x)=d\left(-2x^2\cos(2x)\right)+d\,(2x\sin(2x))-\sin(2x)\,d2x$
$=d\left(-2x^2\cos(2x)+2x\sin(2x)\right)-2\sin(2x)\,dx=d\left(-2x^2\cos(2x)+2x\sin(2x)\right)+d\cos(2x)$
$=d\left(-2x^2\cos(2x)+2x\sin(2x)+\cos(2x)\right)=d\left((-2x^2+1)\cos(2x)+2x\sin(2x)\right)$.

Dus $F(x)=(-2x^2+1)\cos(2x)+2x\sin(2x)+c$.

D6b $(x^2-x+2)e^x\,dx=(x^2-x+2)\,de^x=d\left((x^2-x+2)e^x\right)-e^x\,d\left(x^2-x+2\right)=d\left((x^2-x+2)e^x\right)-(2x-1)e^x\,dx$
$=d\left((x^2-x+2)e^x\right)-(2x-1)\,de^x=d\left((x^2-x+2)e^x\right)-d\left((2x-1)e^x\right)+e^x\,d(2x-1)$
$=d\left((x^2-x+2)e^x-(2x-1)e^x\right)+2e^x\,dx=d\left((x^2-3x+3)e^x\right)+d2e^x$
$=d\left((x^2-3x+5)e^x\right)$. Dus $G(x)=(x^2-3x+5)e^x+c$.

D6c $e^x \sin(2x)\,\mathrm{d}x = \sin(2x)\,\mathrm{d}e^x = \mathrm{d}\left(e^x\sin(2x)\right) - e^x\,\mathrm{d}\sin(2x) = \mathrm{d}\left(e^x\sin(2x)\right) - e^x\cdot 2\cos(2x)\,\mathrm{d}x$

$= \mathrm{d}\left(e^x\sin(2x)\right) - 2\cos(2x)\,\mathrm{d}e^x = \mathrm{d}\left(e^x\sin(2x)\right) - \mathrm{d}\left(2e^x\cos(2x)\right) + e^x\,\mathrm{d}(2\cos(2x))$

$= \mathrm{d}\left(e^x(\sin(2x) - 2\cos(2x))\right) - 4e^x\sin(2x)\,\mathrm{d}x.$

Uit $e^x\sin(2x)\,\mathrm{d}x = \mathrm{d}\left(e^x(\sin(2x)-2\cos(2x))\right) - 4e^x\sin(2x)\,\mathrm{d}x$ volgt nu

$5e^x\sin(2x)\,\mathrm{d}x = \mathrm{d}\left(e^x(\sin(2x)-2\cos(2x))\right) \Rightarrow e^x\sin(2x)\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{5}e^x(\sin(2x)-2\cos(2x))\right)$

Dus $H(x) = \frac{1}{5}e^x(\sin(2x)-2\cos(2x)) + c.$

D7a $\int_{-1}^{1}\frac{2}{x^2+1}\,\mathrm{d}x = \left[2\arctan(x)\right]_{-1}^{1} = 2\arctan(1) - 2\arctan(-1) = 2\cdot\frac{1}{4}\pi - 2\cdot -\frac{1}{4}\pi = \frac{1}{2}\pi + \frac{1}{2}\pi = \pi.$

D7b $\int_{0}^{2\sqrt{3}}\frac{4}{x^2+4}\,\mathrm{d}x = \int_{0}^{2\sqrt{3}}\frac{1}{(\frac{1}{2}x)^2+1}\,\mathrm{d}x = \int_{0}^{2\sqrt{3}}\frac{2}{(\frac{1}{2}x)^2+1}\,\mathrm{d}(\frac{1}{2}x) = \left[2\arctan(\frac{1}{2}x)\right]_{0}^{2\sqrt{3}} = 2\arctan(\sqrt{3}) - 2\arctan(0) = 2\cdot\frac{1}{3}\pi = \frac{2}{3}\pi.$

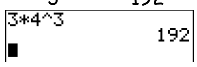
D7c $\int_{1}^{2}\frac{1}{x^2-2x+2}\,\mathrm{d}x = \int_{1}^{2}\frac{1}{(x-1)^2-1+2}\,\mathrm{d}x = \int_{1}^{2}\frac{1}{(x-1)^2+1}\,\mathrm{d}(x-1) = \left[\arctan(x-1)\right]_{1}^{2} = \arctan(1) - \arctan(0) = \frac{1}{4}\pi.$

D7d $\int_{-1}^{1}\frac{3x^2}{x^6+1}\,\mathrm{d}x = \int_{-1}^{1}\frac{1}{(x^3)^2+1}\,\mathrm{d}\left(x^3\right) = \left[\arctan(x^3)\right]_{-1}^{1} = \arctan(1) - \arctan(-1) = \frac{1}{4}\pi - -\frac{1}{4}\pi = \frac{1}{4}\pi + \frac{1}{4}\pi = \frac{1}{2}\pi.$

D7e $\int_{0}^{\frac{1}{2}\pi}\frac{\cos(x)}{\sin^2(x)+1}\,\mathrm{d}x = \int_{0}^{\frac{1}{2}\pi} -\frac{1}{\sin^2(x)+1}\,\mathrm{d}(\sin(x)) = \left[\arctan(\sin(x))\right]_{0}^{\frac{1}{2}\pi}$

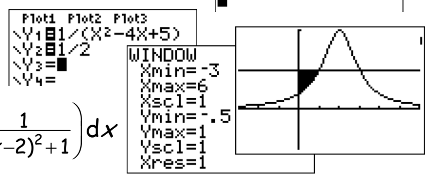
$= \arctan\left(\sin(\frac{1}{2}\pi)\right) - \arctan(\sin(0)) = \arctan(1) - \arctan(0) = \frac{1}{4}\pi.$

D7f $\int_{0}^{1}\frac{\arctan^2(x)}{x^2+1}\,\mathrm{d}x = \int_{0}^{1}\arctan^2(x)\,\mathrm{d}\arctan(x) = \left[\frac{1}{3}\arctan^3(x)\right]_{0}^{1} = \frac{1}{3}\arctan^3(1) - \frac{1}{3}\arctan^3(0) = \frac{1}{3}\cdot(\frac{1}{4}\pi)^3 - \frac{1}{3}\cdot 0^3 = \frac{1}{192}\pi^3.$



D8a $f(0) = \frac{1}{2} \Rightarrow \frac{1}{x^2-4x+5} = \frac{1}{2} \Rightarrow$

$x^2-4x+5 = 2 \Rightarrow x^2-4x+3 = 0 \Rightarrow (x-1)(x-3) = 0 \Rightarrow x = 1 \vee x = 3.$



$O(V) = \int_{0}^{1}\left(\frac{1}{2} - f(x)\right)\mathrm{d}x = \int_{0}^{1}\left(\frac{1}{2} - \frac{1}{x^2-4x+5}\right)\mathrm{d}x = \int_{0}^{1}\left(\frac{1}{2} - \frac{1}{(x-2)^2-4+5}\right)\mathrm{d}x = \int_{0}^{1}\left(\frac{1}{2} - \frac{1}{(x-2)^2+1}\right)\mathrm{d}x$

$= \left[\frac{1}{2}x - \arctan(x-2)\right]_{0}^{1} = \frac{1}{2}\cdot 1 - \arctan(-1) - \left(\frac{1}{2}\cdot 0 - \arctan(-2)\right) = \frac{1}{2} - -\frac{1}{4}\pi + \arctan(-2) = \frac{1}{2} + \frac{1}{4}\pi + \arctan(-2).$

D8b $O(W) = \int_{2}^{p} f(x)\,\mathrm{d}x = \left[\arctan(x-2)\right]_{2}^{p} = \arctan(p-2) = \frac{1}{3}\pi \Rightarrow p-2 = \tan(\frac{1}{3}\pi) = \sqrt{3} \Rightarrow p = 2+\sqrt{3}.$

D9a $\frac{1}{\sqrt{1-4x^2}}\,\mathrm{d}x = \frac{1}{2}\cdot\frac{1}{\sqrt{1-(2x)^2}}\,\mathrm{d}2x = \mathrm{d}\left(\frac{1}{2}\cdot\arcsin(2x)\right).$ Dus $F(x) = \frac{1}{2}\arcsin(2x) + c.$

D9b $\frac{5x^4}{\sqrt{1-x^{10}}}\,\mathrm{d}x = \frac{1}{\sqrt{1-(x^5)^2}}\,\mathrm{d}\left(x^5\right) = \mathrm{d}\left(\arcsin(x^5)\right).$ Dus $G(x) = \arcsin(x^5) + c.$

D9c $\arcsin(2x)\,\mathrm{d}x = \mathrm{d}(x\arcsin(2x)) - x\,\mathrm{d}\arcsin(2x) = \mathrm{d}(x\arcsin(2x)) - x\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2\mathrm{d}x$

$= \mathrm{d}(x\arcsin(2x)) - \frac{2x}{\sqrt{1-4x^2}}\,\mathrm{d}x = \mathrm{d}(x\arcsin(2x)) + \frac{1}{4}\cdot\frac{1}{\sqrt{1-4x^2}}\,\mathrm{d}(1-4x^2) = \mathrm{d}(x\arcsin(2x)) + \frac{1}{2}\cdot\frac{1}{2\sqrt{1-4x^2}}\,\mathrm{d}(1-4x^2)$

$= \mathrm{d}(x\arcsin(2x)) + \mathrm{d}\frac{1}{2}\cdot\sqrt{1-4x^2} = \mathrm{d}\left(x\arcsin(2x) + \frac{1}{2}\cdot\sqrt{1-4x^2}\right).$ Dus $H(x) = x\arcsin(2x) + \frac{1}{2}\sqrt{1-4x^2} + c.$

D9d $\frac{\arcsin^2(x)}{\sqrt{1-x^2}}\,\mathrm{d}x = \arcsin^2(x)\,\mathrm{d}\arcsin(x) = \mathrm{d}\left(\frac{1}{3}\arcsin^3(x)\right).$ Dus $K(x) = \frac{1}{3}\arcsin^3(x) + c.$

$x+4/3x+4\backslash 3$
$3x+12$ −
-8

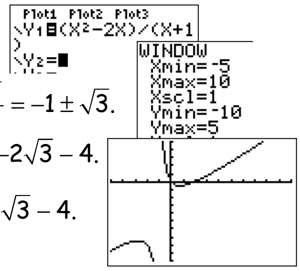
D10a $\int_{-1}^{1}\frac{3x+4}{x+4}\,\mathrm{d}x = \int_{-1}^{1} 3 - \frac{8}{x+4}\,\mathrm{d}x = \left[3x - 8\ln|x+4|\right]_{-1}^{1} = 3 - 8\ln(5) - (-3 - 8\ln(3)) = 6 + 8\ln(3) - 8\ln(5) = 6 + 8\ln(\frac{3}{5}).$

$2x-3/x^2-2x+4\backslash \frac{1}{2}x - \frac{1}{4}$
$x^2 - 1\frac{1}{2}x$ −
$-\frac{1}{2}x + 4$
$-\frac{1}{2}x + \frac{3}{4}$ −
$3\frac{1}{4}$

D10b $\int_{2}^{3}\frac{x^2-2x+4}{2x-3}\,\mathrm{d}x = \int_{2}^{3}\left(\frac{1}{2}x - \frac{1}{4} + \frac{3\frac{1}{4}}{2x-3}\right)\mathrm{d}x = \left[\frac{1}{4}x^2 - \frac{1}{4}x + \frac{13}{8}\ln|2x-3|\right]_{2}^{3}$

$= \frac{9}{4} - \frac{3}{4} + \frac{13}{8}\ln(3) - \left(\frac{4}{4} - \frac{2}{4} + \frac{13}{8}\ln(1)\right) = \frac{6}{4} + \frac{13}{8}\ln(3) - \frac{2}{4} = 1 + \frac{13}{8}\ln(3).$

D11a Noemer $= 0 \Rightarrow x+1=0 \Rightarrow x=-1$. Dus verticale asymptoot: $x=-1$.



$f(x)=\frac{x^2-2x}{x+1} \Rightarrow f'(x)=\frac{(x+1)\cdot(2x-2)-(x^2-2x)\cdot 1}{(x+1)^2}=\frac{2x^2-2x+2x-2-x^2+2x}{(x+1)^2}=\frac{x^2+2x-2}{(x+1)^2}.$

$f'(x)=0$ (teller = 0) $\Rightarrow x^2+2x-2=0$ $(D=2^2-4\cdot 1\cdot -2=4+8=12) \Rightarrow x=\frac{-2\pm\sqrt{12}}{2\cdot 1}=\frac{-2\pm 2\sqrt{3}}{2}=-1\pm\sqrt{3}.$

Max.: (zie plot): $f(-1-\sqrt{3})=\frac{(-1-\sqrt{3})^2+2+2\sqrt{3}}{-1-\sqrt{3}+1}=\frac{1+2\sqrt{3}+3+2+2\sqrt{3}}{-\sqrt{3}}=\frac{6+4\sqrt{3}}{-\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}+12}{-3}=-2\sqrt{3}-4.$

Min.: (zie plot): $f(-1+\sqrt{3})=\frac{(-1+\sqrt{3})^2+2-2\sqrt{3}}{-1+\sqrt{3}+1}=\frac{1-2\sqrt{3}+3+2-2\sqrt{3}}{\sqrt{3}}=\frac{6-4\sqrt{3}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}-12}{3}=2\sqrt{3}-4.$

D11b $f(x)=p$ heeft geen oplossingen (zie plot en D11a) voor $-2\sqrt{3}-4<x<2\sqrt{3}-4$.

D11c $f(x)=0 \Rightarrow \frac{x^2-2x}{x+1}=0$ (teller = 0) $\Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0 \Rightarrow x=0 \vee x=2.$

$x+1/-x^2+2x \quad \backslash -x+3$
$-x^2-x$ −
$3x$
$3x+3$ −
-3

$O(V)=\int_0^2(-f(x))\,dx=\int_0^2\left(-x+3-\frac{3}{x+1}\right)dx=\left[-\frac{1}{2}x^2+3x-3\ln|x+1|\right]_0^2$

$=-\frac{4}{2}+6-3\ln(3)-(0-3\ln(1))=4-3\ln(3).$

D12a $f(x)=\frac{x^2-4x}{x^2+1}=1-\frac{4x+1}{x^2+1}=1-\frac{4x}{x^2+1}-\frac{1}{x^2+1} \Rightarrow F(x)=x-2\ln(x^2+1)-\arctan(x)+c.$

$x^2+1/x^2-4x \quad \backslash 1$
$x^2 \quad +1$ −
$-4x-1$

Want $\frac{4x}{x^2+1}dx=\frac{2}{x^2+1}d(x^2+1)=2\cdot d\ln(x^2+1)=d\,2\ln(x^2+1).$

D12b $g(x)=\frac{x^2+4x}{x^2-4x+4}=1+\frac{8x-4}{(x-2)^2}=1+\frac{8(x-2)+12}{(x-2)^2}=1+\frac{8(x-2)}{(x-2)^2}+\frac{12}{(x-2)^2}=1+\frac{8}{x-2}+12\cdot(x-2)^{-2}.$

$x^2-4x+4/x^2+4x \quad \backslash 1$
x^2-4x+4 −
$8x-4$

Dus $G(x)=x+8\ln|x-2|-12\cdot(x-2)^{-1}+c=x+8\ln|x-2|-\frac{12}{x-2}+c.$

D12c $h(x)=\frac{2x-3}{x^2+6x+10}=\frac{2x+6-9}{x^2+6x+10}=\frac{2x+6}{x^2+6x+10}-\frac{9}{(x+3)^2+1} \Rightarrow H(x)=\ln(x^2+6x+10)-9\arctan(x+3)+c.$

Want $\frac{2x+6}{x^2+6x+10}dx=\frac{1}{x^2+6x+10}d\left(x^2+6x+10\right)$ en $\frac{9}{(x+3)^2+1}dx=\frac{9}{(x+3)^2+1}d(x+3).$

D12d $k(x)=\frac{x^4}{x^2+9}=x^2-9+\frac{81}{x^2+9}=x^2-9+\frac{9}{\frac{x^2}{9}+1}=x^2-9+\frac{9}{(\frac{x}{3})^2+1}.$

$x^2+9/x^4 \quad \backslash x^2-9$
x^4+9x^2 −
$-9x^2$
$-9x^2-81$ −
81

Dus $K(x)=\frac{1}{3}x^3-9x+\frac{9}{\frac{1}{3}}\arctan(\frac{x}{3}+1)+c=\frac{1}{3}x^3-9x+27\arctan(\frac{x}{3})+c.$

D13a $\frac{6}{x^2-1}=\frac{6}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}=\frac{A(x-1)}{(x+1)(x-1)}+\frac{B(x+1)}{(x+1)(x-1)}=\frac{Ax-A+Bx+B}{(x+1)(x-1)}=\frac{(A+B)x-A+B}{(x+1)(x-1)}.$

$\begin{cases}A+B=0\\ -A+B=6\end{cases}$ + $\Rightarrow \begin{cases}B=3\\ A+B=0\end{cases} \Rightarrow A+3=0 \Rightarrow A=-3.$

$2B=6$

$\int_2^4\frac{6}{x^2-1}dx=\int_2^4\left(\frac{-3}{x+1}+\frac{3}{x-1}\right)dx=\left[-3\ln|x+1|+3\ln|x-1|\right]_2^4=-3\ln(5)+3\ln(3)-(-3\ln(3)+3\ln(1))=6\ln(3)-3\ln(5).$

D13b $\frac{3x+2}{x^2+4x+3}=\frac{3x+2}{(x+3)(x+1)}=\frac{A}{x+3}+\frac{B}{x+1}=\frac{A(x+1)}{(x+3)(x+1)}+\frac{B(x+3)}{(x+3)(x+1)}=\frac{Ax+A+Bx+3B}{(x+3)(x+1)}=\frac{(A+B)x+A+3B}{(x+3)(x+1)}.$

$\begin{cases}A+B=3\\ A+3B=2\end{cases}$ − $\Rightarrow \begin{cases}B=-\frac{1}{2}\\ A+B=3\end{cases} \Rightarrow A-\frac{1}{2}=3 \Rightarrow A=3\frac{1}{2}.$

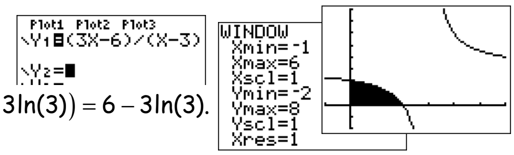
$-2B=1$

$\int_0^1\frac{3x+2}{x^2+4x+3}dx=\int_0^1\left(\frac{3\frac{1}{2}}{x+3}-\frac{\frac{1}{2}}{x+1}\right)dx=\left[3\tfrac{1}{2}\ln|x+3|-\tfrac{1}{2}\ln|x+1|\right]_0^1=3\tfrac{1}{2}\ln(4)-\tfrac{1}{2}\ln(2)-\left(3\tfrac{1}{2}\ln(3)-\tfrac{1}{2}\ln(1)\right)$

$=3\tfrac{1}{2}\ln(2^2)-\tfrac{1}{2}\ln(2)-3\tfrac{1}{2}\ln(3)=7\ln(2)-\tfrac{1}{2}\ln(2)-3\tfrac{1}{2}\ln(3)=6\tfrac{1}{2}\ln(2)-3\tfrac{1}{2}\ln(3).$

D14a $f(x)=0$ (teller = 0) $\Rightarrow 3x-6=0 \Rightarrow 3x=6 \Rightarrow x=2.$

$x-3/3x-6 \quad \backslash 3$
$3x-9$ −
3



$O(V)=\int_0^2 f(x)\,dx=\int_0^2\left(3+\frac{3}{x-3}\right)dx=\left[3x+3\ln|x-3|\right]_0^2=6+3\ln(1)-(0+3\ln(3))=6-3\ln(3).$

D14b $I(L)=\int_0^2\pi(f(x))^2dx=\int_0^2\pi\left(\frac{3x-6}{x-3}\right)^2dx=\int_0^2\pi\left(\frac{9x^2-36x+36}{x^2-6x+9}\right)dx=\int_0^2\pi\left(9+\frac{18x-45}{x^2-6x+9}\right)dx$

$x^2-6x+9/9x^2-36x+36 \quad \backslash 9$
$9x^2-54x+81$ −
$18x-45$

$=\int_0^2\pi\left(9+\frac{18(x-3)+9}{(x-3)^2}\right)dx=\int_0^2\pi\left(9+\frac{18}{x-3}+9(x-3)^{-2}\right)dx=\left[\pi\left(9x+18\ln|x-3|-9(x-3)^{-1}\right)\right]_0^2$

$=\left[\pi\left(9x+18\ln|x-3|-\frac{9}{x-3}\right)\right]_0^2=\pi(18+18\ln(1)+9)-\pi(0+18\ln(3)+3)=27\pi-18\ln(3)-3\pi=24\pi-18\ln(3).$

Gemengde opgaven K. Voortgezette integraalrekening

G39a $f(x)=\frac{x+2}{x^2-4x+8}=\frac{\frac{1}{2}\cdot(2x-4)+4}{x^2-4x+8}=\frac{1}{2}\cdot\frac{2x-4}{x^2-4x+8}+\frac{4}{(x-2)^2+4}\Rightarrow F(x)=\frac{1}{2}\ln\left|x^2-4x+8\right|+2\arctan(\frac{1}{2}x-1)+c.$

Want $\frac{1}{2}\cdot\frac{2x-4}{x^2-4x+8}dx=\frac{1}{2}\cdot\frac{1}{x^2-4x+8}d(x^2-4x+8)=\frac{1}{2}\cdot d\ln\left|x^2-4x+8\right|=d\,\frac{1}{2}\ln\left|x^2-4x+8\right|$

en $\frac{4}{(x-2)^2+4}dx=\frac{1}{\frac{1}{4}\cdot(x-2)^2+1}dx=\frac{1}{(\frac{1}{2}x-1)^2+1}dx=\frac{2}{(\frac{1}{2}x-1)^2+1}d(\frac{1}{2}x-1)=d\,2\arctan(\frac{1}{2}x-1).$

G39b $3x^2\cdot\sin(x^3)dx=\sin(x^3)d(x^3)=d\left(-\cos(x^3)\right)$ dus $f(x)=3x^2\cdot\sin(x^3)\Rightarrow F(x)=-\cos(x^3)+c.$

G39c $\cos(x)\cdot\sqrt[3]{1+\sin(x)}\,dx=(1+\sin(x))^{\frac{1}{3}}\,d(1+\sin(x))=d\left(\frac{1}{\frac{4}{3}}\cdot(1+\sin(x))^{\frac{4}{3}}\right)=d\left(\frac{3}{4}\cdot(1+\sin(x))\cdot\sqrt[3]{1+\sin(x)}\right).$

$f(x)=\cos(x)\cdot\sqrt[3]{1+\sin(x)}\Rightarrow F(x)=\frac{3}{4}\cdot(1+\sin(x))\cdot\sqrt[3]{1+\sin(x)}+c.$

G39d $(2x+4)\cdot\cos(2x)dx=\frac{1}{2}(2x+4)d\sin(2x)=(x+2)d\sin(2x)=d((x+2)\cdot\sin(2x))-\sin(2x)d(x+2)$

$=d((x+2)\cdot\sin(2x))-\sin(2x)dx=d((x+2)\cdot\sin(2x))+d\left(\frac{1}{2}\cos(2x)\right).$

$f(x)=(2x+4)\cdot\cos(2x)\Rightarrow F(x)=(x+2)\cdot\sin(2x)+\frac{1}{2}\cos(2x)+c.$

G39e $\frac{2x+1}{\sqrt{16-x^2}}dx=\frac{2x}{\sqrt{16-x^2}}dx+\frac{1}{\sqrt{16-x^2}}dx=-\frac{1}{\sqrt{16-x^2}}d(16-x^2)+\frac{\frac{1}{4}}{\sqrt{1-\frac{1}{16}x^2}}dx=-2\cdot\frac{1}{2\sqrt{16-x^2}}d(16-x^2)+\frac{1}{\sqrt{1-(\frac{x}{4})^2}}d(\frac{x}{4})$

$=-2\,d(\sqrt{16-x^2})+d\left(\arctan(\frac{1}{4}x)\right).$ Dus $f(x)=\frac{2x+1}{\sqrt{16-x^2}}\Rightarrow F(x)=-2\sqrt{16-x^2}+\arctan(\frac{1}{4}x)+c.$

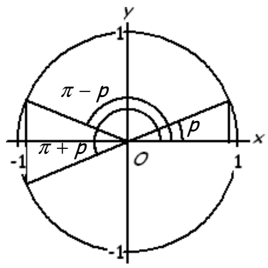
G39f $\frac{2x+4}{\sqrt{-x^2-4x-3}}dx=-\frac{1}{\sqrt{-x^2-4x-3}}d(-x^2-4x-3)=-2\frac{1}{2\sqrt{-x^2-4x-3}}d(-x^2-4x-3)=d(-2\sqrt{-x^2-4x-3}).$

$f(x)=\frac{2x+4}{\sqrt{-x^2-4x-3}}\Rightarrow F(x)=-2\sqrt{-x^2-4x-3}+c.$

G39g $f(x)=\frac{2x+7}{x^2+6x+8}=\frac{2x+7}{(x+4)(x+2)}=\frac{A}{x+4}+\frac{B}{x+2}=\frac{A(x+2)}{(x+4)(x+2)}+\frac{B(x+4)}{(x+4)(x+2)}=\frac{Ax+2A+Bx+4B}{(x+4)(x+2)}=\frac{(A+B)x+2A+4B}{(x+4)(x+2)}.$

$\begin{cases}A+B=2 & \times 2\\ 2A+4B=7 & \times 1\end{cases}\Rightarrow\begin{cases}2A+2B=4\\ 2A+4B=7\end{cases}\;+\;\;-2B=-3\;\Rightarrow\begin{cases}B=1\frac{1}{2}\\ A+B=2\end{cases}\Rightarrow A+1\frac{1}{2}=2\Rightarrow A=\frac{1}{2}.$

$f(x)=\frac{2x+7}{x^2+6x+8}\Rightarrow F(x)=\frac{1}{2}\ln|x+4|+1\frac{1}{2}\ln|x+2|+c.$



G40a $f(\pi+p)=4\sin^2(\pi+p)\cos(\pi+p)=4(-\sin(p))^2\cdot-\cos(p)=-4\sin^2(p)\cos(p).$

$f(\pi-p)=4\sin^2(\pi-p)\cos(\pi-p)=4\sin^2(p)\cdot-\cos(p)=-4\sin^2(p)\cos(p).$

Voor elke p geldt: $f(\pi+p)=f(\pi-p)\Rightarrow$ de lijn $x=\pi$ is symmetrieas van de grafiek van f.

G40b $f(x)=4\sin^2(x)\cos(x)\Rightarrow f'(x)=4\cdot2\sin(x)\cos(x)\cdot\cos(x)+4\sin^2(x)\cdot-\sin(x)=8\sin(x)\cos^2(x)-4\sin^3(x).$

$f(\pi)=4\sin^2(\pi)\cos(\pi)=4\cdot0^2\cdot-1=0$ en $f'(\pi)=8\sin(\pi)\cos^2(\pi)-4\sin^3(\pi)=8\cdot0\cdot(-1)^2-4\cdot0^3=0.$

Dus de grafiek van f raakt de x-as in $(\pi, 0)$.

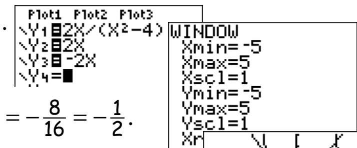
G40c $f(\frac{1}{2}\pi)=4\sin^2(\frac{1}{2}\pi)\cos(\frac{1}{2}\pi)=0\Rightarrow A(\frac{1}{2}\pi,0)$ en $f'(\frac{1}{2}\pi)=8\sin(\frac{1}{2}\pi)\cos^2(\frac{1}{2}\pi)-4\sin^3(\frac{1}{2}\pi)=8\cdot1\cdot0^2-4\cdot1^3=-4.$

Raaklijn: $y=-4x+b$ door $A(\frac{1}{2}\pi,0)\Rightarrow0=-4\cdot\frac{1}{2}\pi+b\Rightarrow2\pi=b$. Dus raaklijn $y=-4x+2\pi$.

G40d $f(x)dx=4\sin^2(x)\cos(x)dx=4\sin^2(x)d\sin(x)=d\left(\frac{4}{3}\sin^3(x)\right)\Rightarrow F(x)=\frac{4}{3}\sin^3(x)+c.$

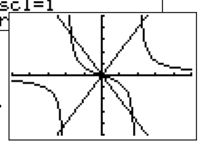
$$O=2\cdot\int_0^{\frac{1}{2}\pi}f(x)dx+2\cdot\int_{\frac{1}{2}\pi}^{\pi}(-f(x))dx=2\left[\frac{4}{3}\sin^3(x)\right]_0^{\frac{1}{2}\pi}+2\left[-\frac{4}{3}\sin^3(x)\right]_{\frac{1}{2}\pi}^{\pi}$$

$$=\frac{8}{3}\sin^3(\frac{1}{2}\pi)-\frac{8}{3}\sin^3(0)-\left(\frac{8}{3}\sin^3(\pi)-\frac{8}{3}\sin^3(\frac{1}{2}\pi)\right)=\frac{8}{3}-0-\left(0-\frac{8}{3}\right)=\frac{8}{3}+\frac{8}{3}=\frac{16}{3}=5\frac{1}{3}.$$



G41a $f(x)=\frac{2x}{x^2-4}\Rightarrow f'(x)=\frac{(x^2-4)\cdot2-2x\cdot2x}{(x^2-4)^2}=\frac{2x^2-8-4x^2}{(x^2-4)^2}=\frac{-2x^2-8}{(x^2-4)^2}.$ Dus $f'(0)=\frac{-8}{(-4)^2}=-\frac{8}{16}=-\frac{1}{2}.$

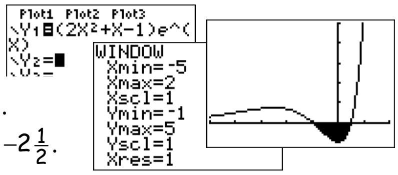
De lijn $y=ax$ snijdt de grafiek van f in precies drie punten (zie plot) voor $a<-\frac{1}{2}\vee a>0.$



G41b $\frac{2x}{x^2-4}=\frac{2}{3}\Rightarrow2x^2-8=6x\Rightarrow2x^2-6x-8=0\Rightarrow x^2-3x-4=0\Rightarrow(x-4)(x+1)=0\Rightarrow x=4\vee x=-1.$

$$O=\int_{-1}^{0}\left(\frac{2}{3}-f(x)\right)dx=\int_{-1}^{0}\left(\frac{2}{3}-\frac{2x}{x^2-4}\right)dx=\left[\frac{2}{3}x-\ln\left|x^2-4\right|\right]_{-1}^{0}=0-\ln(4)-\left(-\frac{2}{3}-\ln(3)\right)=\frac{2}{3}+\ln(3)-\ln(4)=\frac{2}{3}+\ln(\frac{3}{4}).$$

Want $\frac{2x}{x^2-4}dx=\frac{1}{x^2-4}d(x^2-4)=d\ln\left|x^2-4\right|\Rightarrow F(x)=\ln\left|x^2-4\right|+c.$



G42a $f(x)=(2x^2+x-1)e^x \Rightarrow f'(x)=(4x+1)e^x+(2x^2+x-1)e^x=(2x^2+5x)e^x$.

$f'(x)=0 \Rightarrow (2x^2+5x)e^x=0 \Rightarrow 2x^2+5x=0 \Rightarrow x(2x+5)=0 \Rightarrow x=0 \vee x=-2\frac{1}{2}$.

$f(-2\frac{1}{2})=(2\cdot 6\frac{1}{4}-2\frac{1}{2}-1)e^{-2\frac{1}{2}}=\frac{12\frac{1}{2}-2\frac{1}{2}-1}{e^{2\frac{1}{2}}}=\frac{9}{e^2\cdot\sqrt{e}}$ en $f(0)=-1\cdot e^0=-1\cdot 1=-1 \Rightarrow$ toppen $(-2\frac{1}{2},\frac{9}{e^2\cdot\sqrt{e}})$ en $(0,-1)$.

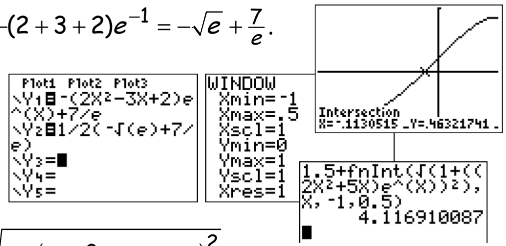
G42b $f(x)=(2x^2+x-1)e^x=0 \Rightarrow 2x^2+x-1=0$ $(D=1^2-4\cdot 2\cdot -1=1+8=9) \Rightarrow x=\frac{-1-3}{2\cdot 2}=-1 \vee x=\frac{-1+3}{4}=\frac{1}{2}$.

$(2x^2+x-1)e^x\,dx=(2x^2+x-1)\,de^x=d\left((2x^2+x-1)e^x\right)-e^x d(2x^2+x-1)=d\left((2x^2+x-1)e^x\right)-(4x+1)e^x\,dx$

$=d\left((2x^2+x-1)e^x\right)-(4x+1)\,de^x=d\left((2x^2+x-1)e^x\right)-\left(d\left((4x+1)e^x\right)-e^x d(4x+1)\right)$

$=d\left((2x^2+x-1)e^x\right)-d\left((4x+1)e^x\right)+4e^x\,dx=d\left((2x^2+x-1)e^x-(4x+1)e^x+4e^x\right)=d\left((2x^2-3x+2)e^x\right)$.

$O(V)=\int_{-1}^{\frac{1}{2}}(-f(x))\,dx=\left[-(2x^2-3x+2)e^x\right]_{-1}^{\frac{1}{2}}=-(\frac{2}{4}-\frac{3}{2}+2)e^{\frac{1}{2}}--(2+3+2)e^{-1}=-\sqrt{e}+\frac{7}{e}$.



G42c $\int_{-1}^{p}(-f(x))\,dx=\frac{1}{2}O(V) \Rightarrow \left[-(2x^2-3x+2)e^x\right]_{-1}^{p}=\frac{1}{2}\left(-\sqrt{e}+\frac{7}{e}\right) \Rightarrow$

$-(2p^2-3p+2)e^p+\frac{7}{e}=\frac{1}{2}\left(-\sqrt{e}+\frac{7}{e}\right)$ (intersect) $\Rightarrow p\approx -0{,}113$.

G42d omtrek$(V)=1\frac{1}{2}+$ booglengte $=1\frac{1}{2}+\int_{-1}^{\frac{1}{2}}\sqrt{1+\left(f'(x)\right)^2}\,dx=1\frac{1}{2}+\int_{-1}^{\frac{1}{2}}\sqrt{1+\left((2x^2+5x)e^x\right)^2}\,dx$ (fnInt) $\approx 4{,}117$.

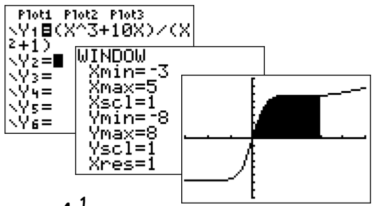
G43a $f(x)=\frac{x^3+10x}{x^2+1} \Rightarrow f'(x)=\frac{(x^2+1)\cdot(3x^2+10)-(x^3+10x)\cdot 2x}{(x^2+1)^2}=\frac{3x^4+10x^2+3x^2+10-2x^4-20x^2}{(x^2+1)^2}=\frac{x^4-7x^2+10}{(x^2+1)^2}$.

$f'(x)=0$ (teller = 0) $\Rightarrow x^4-7x^2+10=0 \Rightarrow (x^2-2)(x^2-5)=0 \Rightarrow x^2=2 \vee x^2=5 \Rightarrow x^2=\pm\sqrt{2} \vee x=\pm\sqrt{5}$.

$f(-\sqrt{2})=\frac{-2\sqrt{2}-10\sqrt{2}}{2+1}=\frac{-12\sqrt{2}}{3}=-4\sqrt{2}$ en $f(\sqrt{2})=\frac{2\sqrt{2}+10\sqrt{2}}{2+1}=\frac{12\sqrt{2}}{3}=4\sqrt{2}$.

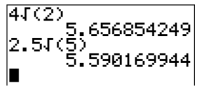
$f(-\sqrt{5})=\frac{-5\sqrt{5}-10\sqrt{5}}{5+1}=\frac{-15\sqrt{5}}{6}=-2\frac{1}{2}\sqrt{5}$ en $f(\sqrt{5})=\frac{5\sqrt{5}+10\sqrt{5}}{5+1}=\frac{15\sqrt{5}}{6}=2\frac{1}{2}\sqrt{5}$.

De toppen zijn $(-\sqrt{5},\,-2\frac{1}{2}\sqrt{5})$, $(-\sqrt{2},\,-4\sqrt{2})$, $(\sqrt{2},4\sqrt{2})$, en $(\sqrt{5},2\frac{1}{2}\sqrt{5})$.



G43b $f(1)=\frac{1+10}{1+1}=\frac{11}{2}=5\frac{1}{2} \Rightarrow A(1,5\frac{1}{2})$ en $f'(1)=\frac{1-7+10}{(1+1)^2}=\frac{4}{4}=1$.

Raaklijn: $y=x+b$ door $A(1,5\frac{1}{2}) \Rightarrow 5\frac{1}{2}=1+b \Rightarrow 4\frac{1}{2}=b$. Dus de raaklijn is: $y=x+4\frac{1}{2}$.



G43c $f(x)=p$ heeft precies een oplossing (zie plot) voor $p<-4\sqrt{2} \vee -2\frac{1}{2}\sqrt{5}<p<2\frac{1}{2}\sqrt{5} \vee p>4\sqrt{2}$.

G43d $\frac{x^3+10x}{x^2+1}dx=x\,dx+\frac{9x}{x^2+1}dx=d\left(\frac{1}{2}x^2\right)+\frac{4\frac{1}{2}}{x^2+1}d(x^2+1)=d\left(\frac{1}{2}x^2\right)+4\frac{1}{2}\,d\ln(x^2+1)=d\left(\frac{1}{2}x^2+4\frac{1}{2}\ln(x^2+1)\right)$.

$O=\int_0^3 f(x)\,dx=\left[\frac{1}{2}x^2+4\frac{1}{2}\ln(x^2+1)\right]_0^3=\frac{9}{2}+4\frac{1}{2}\ln(10)-\left(0+4\frac{1}{2}\ln(1)\right)=4\frac{1}{2}+4\frac{1}{2}\ln(10)$.

$x^2+1\,/\,x^3+10x\ \backslash\ x$
$\underline{x^3+\ \ x}\ -$
$9x$

G44a $x^3\ln(x)dx=\ln(x)d\left(\frac{1}{4}x^4\right)=d\left(\frac{1}{4}x^4\ln(x)\right)-\frac{1}{4}x^4 d\ln(x)=d\left(\frac{1}{4}x^4\ln(x)\right)-\frac{1}{4}x^4\cdot\frac{1}{x}dx$

$=d\left(\frac{1}{4}x^4\ln(x)\right)-\frac{1}{4}x^3dx=d\left(\frac{1}{4}x^4\ln(x)\right)-d\left(\frac{1}{16}x^4\right)=d\left(\frac{1}{4}x^4\ln(x)-\frac{1}{16}x^4\right)$.

$f(x)=x^3\ln(x) \Rightarrow F(x)=\frac{1}{4}x^4\ln(x)-\frac{1}{16}x^4+c$.

G44b $x\ln(x^3)dx=x\cdot 3\ln(x)dx=3\ln(x)d\left(\frac{1}{2}x^2\right)=d\left(1\frac{1}{2}x^2\ln(x)\right)-1\frac{1}{2}x^2 d\ln(x)=d\left(1\frac{1}{2}x^2\ln(x)\right)-1\frac{1}{2}x^2\cdot\frac{1}{x}dx$

$=d\left(1\frac{1}{2}x^2\ln(x)\right)-1\frac{1}{2}x\,dx=d\left(1\frac{1}{2}x^2\ln(x)\right)-d\left(\frac{3}{4}x^2\right)=d\left(1\frac{1}{2}x^2\ln(x)-\frac{3}{4}x^2\right)$.

$f(x)=x\ln(x^3) \Rightarrow F(x)=1\frac{1}{2}x^2\ln(x)-\frac{3}{4}x^2+c$.

G44c $x^n\ln(x)dx=\ln(x)d\left(\frac{1}{n+1}x^{n+1}\right)=d\left(\frac{1}{n+1}x^{n+1}\ln(x)\right)-\frac{1}{n+1}x^{n+1}d\ln(x)=d\left(\frac{1}{n+1}x^{n+1}\ln(x)\right)-\frac{1}{n+1}x^{n+1}\cdot\frac{1}{x}dx$

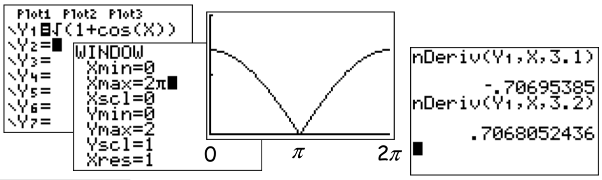
$=d\left(\frac{1}{n+1}x^{n+1}\ln(x)\right)-\frac{1}{n+1}x^n dx=d\left(\frac{1}{n+1}x^{n+1}\ln(x)\right)-d\left(\frac{1}{(n+1)^2}x^{n+1}\right)=d\left(\frac{1}{n+1}x^{n+1}\ln(x)-\frac{1}{(n+1)^2}x^{n+1}\right)$.

$f_n(x)=x^n\ln(x) \Rightarrow F(x)=\frac{1}{n+1}x^{n+1}\ln(x)-\frac{1}{(n+1)^2}x^{n+1}+c$.

G44d $x\ln(x^m)dx=x\cdot m\ln(x)dx=m\ln(x)d\left(\frac{1}{2}x^2\right)=d\left(\frac{1}{2}mx^2\ln(x)\right)-\frac{1}{2}mx^2 d\ln(x)=d\left(\frac{1}{2}mx^2\ln(x)\right)-\frac{1}{2}mx^2\cdot\frac{1}{x}dx$

$=d\left(\frac{1}{2}mx^2\ln(x)\right)-\frac{1}{2}xm\,dx=d\left(\frac{1}{2}mx^2\ln(x)\right)-d\frac{1}{4}mx^2=d\left(\frac{1}{2}mx^2\ln(x)-\frac{1}{4}mx^2\right)$.

$f_m(x)=x\ln(x^m) \Rightarrow F(x)=\frac{1}{2}mx^2\ln(x)-\frac{1}{4}mx^2+c$.



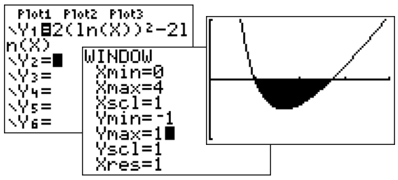
G45a Maak een schets van de plot hiernaast.

G45b $f'(3{,}1) \approx -0{,}707$ en $f'(3{,}1) \approx 0{,}707$.
Ik denk dat de grafiek de x-as niet raakt. ($f'(\pi) \neq 0$)

G45c $f(x) = \sqrt{1+\cos(x)}$ (gebruik nu: $\cos(2A) = 2\cos^2(A) - 1$) $= \sqrt{1 + 2\cos^2(\frac{1}{2}x) - 1}$
$= \sqrt{2\cos^2(\frac{1}{2}x)} = \sqrt{2} \cdot \sqrt{\cos^2(\frac{1}{2}x)} = \sqrt{2} \cdot \left|\cos(\frac{1}{2}x)\right| = \left|\sqrt{2} \cdot \cos(\frac{1}{2}x)\right|$.
Voor $0 < x < \pi$ is $\cos(\frac{1}{2}x) > 0 \Rightarrow f(x) = \sqrt{2} \cdot \cos(\frac{1}{2}x) \Rightarrow f'(x) = \sqrt{2} \cdot -\sin(\frac{1}{2}x) \cdot \frac{1}{2} = -\frac{1}{2}\sqrt{2} \cdot \sin(\frac{1}{2}x)$.
$\lim\limits_{x \to \pi} f'(x) = -\frac{1}{2}\sqrt{2} \cdot \sin(\frac{1}{2}\pi) = -\frac{1}{2}\sqrt{2} \cdot 1 = -\frac{1}{2}\sqrt{2} \neq 0$, dus de grafiek van f raakt de x-as niet.

G45d $O = \int_0^{\pi} \sqrt{1+\cos(x)}\,dx = \int_0^{\pi} \sqrt{2} \cdot \cos(\frac{1}{2}x)\,dx = \left[2\sqrt{2} \cdot \sin(\frac{1}{2}x)\right]_0^{\pi} = 2\sqrt{2} \cdot \sin(\frac{1}{2}\pi) - 2\sqrt{2} \cdot \sin(0) = 2\sqrt{2} \cdot 1 - 0 = 2\sqrt{2}$.

G46a $f_1(x) = 2\ln^2(x) - 2\ln(x) = 0 \Rightarrow 2\ln(x) \cdot (\ln(x) - 1) = 0 \Rightarrow \ln(x) = 0 \vee \ln(x) = 1 \Rightarrow x = e^0 = 1 \vee x = e^1 = e$.



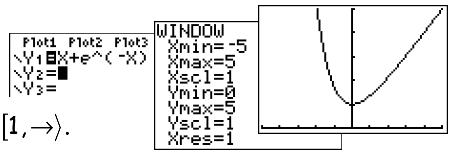
$\left(2\ln^2(x) - 2\ln(x)\right)dx = (2\ln(x) \cdot (\ln(x) - 1))\,dx = 2(\ln(x) - 1)\,d(x\ln(x) - x)$
$= d\,(2(\ln(x) - 1) \cdot (x\ln(x) - x)) - 2 \cdot (x\ln(x) - x)\,d(\ln(x) - 1)$
$= d\,(2(\ln(x) - 1) \cdot (x\ln(x) - x)) - 2 \cdot (x\ln(x) - x) \cdot \frac{1}{x}\,dx$
$= d\,(2(\ln(x) - 1) \cdot (x\ln(x) - x)) - 2 \cdot (\ln(x) - 1)\,dx$
$= d\,(2(\ln(x) - 1) \cdot (x\ln(x) - x)) - d(2 \cdot (x\ln(x) - x - x)$
$= d\,(2(\ln(x) - 1) \cdot (x\ln(x) - x) - 2(x\ln(x) - 2x))$.

$O = \int_1^{e} (-f_1(x))\,dx = \left[-2(\ln(x) - 1) \cdot (x\ln(x) - x) + 2(x\ln(x) - 2x)\right]_1^{e}$
$= -2(\ln(e) - 1) \cdot (e\ln(e) - e) + 2(e\ln(e) - 2e) - (-2(\ln(1) - 1) \cdot (\ln(1) - 1) + 2(\ln(1) - 2))$
$= -2(1 - 1)(e \cdot 1 - e) + 2(e \cdot 1 - 2e) - (-2(0 - 1)(0 - 1) + 2(0 - 2)) = 0 - 2e - (-2 \cdot -1 \cdot -1 - 4) = -2e - (-2 - 4) = -2e + 6$.

G46b $f_p(x) = 2\ln^2(x) - 2p\ln(x) = 0 \Rightarrow 2\ln(x) \cdot (\ln(x) - p) = 0 \Rightarrow \ln(x) = 0 \vee \ln(x) = p \Rightarrow x = e^0 = 1 \vee x = e^p$.
$\left(2\ln^2(x) - 2p\ln(x)\right)dx = (2\ln(x) \cdot (\ln(x) - p))\,dx = 2(\ln(x) - p)\,d(x\ln(x) - x)$
$= d\,(2(\ln(x) - p) \cdot (x\ln(x) - x)) - 2 \cdot (x\ln(x) - x)\,d(\ln(x) - p)$
$= d\,(2(\ln(x) - p) \cdot (x\ln(x) - x)) - 2 \cdot (x\ln(x) - x) \cdot \frac{1}{x}\,dx$
$= d\,(2(\ln(x) - p) \cdot (x\ln(x) - x)) - 2 \cdot (\ln(x) - 1)\,dx$
$= d\,(2(\ln(x) - p) \cdot (x\ln(x) - x)) - d(2 \cdot (x\ln(x) - x - x))$
$= d\,(2(\ln(x) - p) \cdot (x\ln(x) - x) - 2(x\ln(x) + 2x))$.

$O = \int_1^{e^p} \left(-f_p(x)\right)dx = \left[-2(\ln(x) - p) \cdot (x\ln(x) - x) + 2(x\ln(x) - 2x)\right]_1^{e^p}$
$= -2(\ln(e^p) - p) \cdot (e^p\ln(e^p) - e^p) + 2(e^p\ln(e^p) - 2e^p) - (-2(\ln(1) - p) \cdot (\ln(1) - 1) + 2(\ln(1) - 2))$
$= -2(p - p)(e^p \cdot p - e^p) + 2(e^p \cdot p - 2e^p) - (-2(0 - p)(0 - 1) + 2(0 - 2)) = 0 + 2pe^p - 4e^p + 2p + 4$.
$O = 8 \Rightarrow 2pe^p - 4e^p + 2p + 4 = 8 \Rightarrow e^p(2p - 4) + 2p - 4 = 0 \Rightarrow (2p - 4)(e^p + 1) = 0 \Rightarrow p = 2 \vee e^p = -1 \Rightarrow p = 2$.

G47a $f(x) = x + e^{-x} \Rightarrow f'(x) = 1 + e^{-x} \cdot -1 = 1 - e^{-x}$.
$f'(x) = 0 \Rightarrow 1 - e^{-x} = 0 \Rightarrow 1 = e^{-x} \Rightarrow -x = 0 \Rightarrow x = 0$.
Minimum (zie plot) is $f(0) = 0 + e^0 = 1 \Rightarrow B_f$ (zie plot) $= [1, \rightarrow\rangle$.



G47b $O(V_p) = \int_{-p}^{p} f(x)\,dx = \int_{-p}^{p} \left(x + e^{-x}\right)dx = \left[\frac{1}{2}x^2 - e^{-x}\right]_{-p}^{p} = \frac{1}{2}p^2 - e^{-p} - \left(\frac{1}{2}p^2 - e^{p}\right) = e^p - e^{-p}$.
$O(V_p) = 6 \Rightarrow e^p - \frac{1}{e^p} = 6 \Rightarrow e^p + 6 - \frac{1}{e^p} = 0 \Rightarrow (e^p)^2 + 6e^p - 1 = 0$ (stel $e^p = t$) $\Rightarrow t^2 + 6t + 1 = 0$ ($D = 62 - 4 \cdot 1 \cdot -1 = 40$) $\Rightarrow$
$t = \frac{-6 \pm \sqrt{40}}{2 \cdot 1} = \frac{-6 \pm 2\sqrt{10}}{2} = -3 \pm \sqrt{10} \Rightarrow t = e^p = -3 - \sqrt{10}$ (geen oplossing) $\vee$ $t = e^p = -3 + \sqrt{10} \Rightarrow p = \ln(-3 + \sqrt{10})$.

G47c $I(L) = \int_{-1}^{0} \pi(f(x))^2\,dx = \int_{-1}^{0} \pi\left(x + e^{-x}\right)^2 dx = \int_{-1}^{0} \pi\left(x^2 + 2xe^{-x} + e^{-2x}\right)dx = \left[\pi\left(\frac{1}{3}x^3 - 2xe^{-x} - 2e^{-x} - \frac{1}{2}e^{-2x}\right)\right]_{-1}^{0}$
$= \pi\left(0 - 0 - 2e^0 - \frac{1}{2}e^0\right) - \pi\left(-\frac{1}{3} + 2e^1 - 2e^1 - \frac{1}{2}e^2\right) = \pi\left(0 - 0 - 2 - \frac{1}{2}\right) - \pi\left(-\frac{1}{3} - \frac{1}{2}e^2\right) = \left(\frac{1}{2}e^2 - 2\frac{1}{6}\right)\pi$.
$2xe^{-x}dx = -2x\,de^{-x} = d(-2xe^{-x}) - e^{-x}d(-2x) = d(-2xe^{-x}) + 2e^{-x}dx = d(-2xe^{-x}) + d(-2e^{-x}) = d(-2xe^{-x} - 2e^{-x})$.

G48a $f(x)\mathrm{d}x = \frac{2}{\sin(2x)}\mathrm{d}x = \frac{2}{2\sin(x)\cos(x)}\mathrm{d}x = \frac{1}{\sin(x)\cos(x)}\mathrm{d}x = \frac{\frac{1}{\cos^2(x)}}{\frac{\sin(x)\cos(x)}{\cos^2(x)}}\mathrm{d}x = \frac{\frac{1}{\cos^2(x)}}{\frac{\sin(x)}{\cos(x)}}\mathrm{d}x = \frac{\frac{1}{\cos^2(x)}}{\tan(x)}\mathrm{d}x$

$= \frac{2}{\sin(2x)}\mathrm{d}x = \frac{\frac{1}{\cos^2(x)}}{\tan(x)}\mathrm{d}x = \frac{1}{\tan(x)}\mathrm{d}\tan(x) = \mathrm{d}\ln|\tan(x)| \Rightarrow F(x) = \ln|\tan(x)| + c.$

G48b $f(x)\mathrm{d}x = \frac{1}{e^x+1}\mathrm{d}x = \frac{e^{-x}}{1+e^{-x}}\mathrm{d}x = -\frac{1}{1+e^{-x}}\mathrm{d}e^{-x} = -\mathrm{d}\ln\left|1+e^{-x}\right| \Rightarrow F(x) = -\ln(1+e^{-x}) + c.$

G48c $F(x) = (ax^3+bx^2+cx+d)e^{2x} \Rightarrow F'(x) = (3ax^2+2bx+c)e^{2x} + (ax^3+bx^2+cx+d)e^{2x}\cdot 2$

$= \left(2ax^3+(3a+2b)x^2+(2b+2c)x+(c+2d)\right)\cdot e^{2x}.$

$F'(x) = f(x) \Rightarrow \left(2ax^3+(3a+2b)x^2+(2b+2c)x+(c+2d)\right)\cdot e^{2x} = \left(x^3-3x+2\right)\cdot e^{2x}$

$$\Rightarrow \begin{cases} 2a=1 \\ 3a+2b=0 \\ 2b+2c=-3 \\ c+2d=2 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{2} \\ 1\frac{1}{2}+2b=0 \Rightarrow 2b=-1\frac{1}{2} \Rightarrow b=-\frac{3}{4} \\ -1\frac{1}{2}+2c=-3 \Rightarrow 2c=-1\frac{1}{2} \Rightarrow c=-\frac{3}{4} \\ -\frac{3}{4}+2d=2 \Rightarrow 2d=2\frac{3}{4} \Rightarrow d=1\frac{3}{8} \end{cases} \Rightarrow F(x) = (\tfrac{1}{2}x^3-\tfrac{3}{4}x^2-\tfrac{3}{4}x+1\tfrac{3}{8})e^{2x}+c.$$

G48d $f(x)\mathrm{d}x = \frac{\ln(\sin(x))}{\cos^2(x)}\mathrm{d}x = \ln(\sin(x))\mathrm{d}\tan(x) = \mathrm{d}\left(\ln(\sin(x))\cdot\tan(x)\right) - \tan(x)\mathrm{d}\ln(\sin(x))$

$= \mathrm{d}\left(\ln(\sin(x))\cdot\tan(x)\right) - \tan(x)\frac{1}{\sin(x)}\cdot\cos(x)\mathrm{d}x = \mathrm{d}\left(\ln(\sin(x))\cdot\tan(x)\right) - \frac{\sin(x)}{\cos(x)}\frac{1}{\sin(x)}\cdot\cos(x)\mathrm{d}x$

$= \mathrm{d}\left(\ln(\sin(x))\cdot\tan(x) - x\right) \Rightarrow F(x) = \ln(\sin(x))\cdot\tan(x) - x + c.$

G48e $f(x)\mathrm{d}x = \sqrt{1-x^2}\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - x\,\mathrm{d}\sqrt{1-x^2} = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - x\cdot\frac{1}{2\sqrt{1-x^2}}\cdot -2x\,\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - \frac{-x^2}{\sqrt{1-x^2}}\mathrm{d}x$

$= \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - \frac{1-x^2-1}{\sqrt{1-x^2}}\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - \frac{1-x^2}{\sqrt{1-x^2}}\mathrm{d}x + \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - \sqrt{1-x^2}\mathrm{d}x + \mathrm{d}\arcsin(x).$

Uit $\sqrt{1-x^2}\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}\right) - \sqrt{1-x^2}\mathrm{d}x + \mathrm{d}\arcsin(x)$ volgt nu:

$2\cdot\sqrt{1-x^2}\mathrm{d}x = \mathrm{d}\left(x\cdot\sqrt{1-x^2}+\arcsin(x)\right) \Rightarrow \sqrt{1-x^2}\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}x\cdot\sqrt{1-x^2}+\frac{1}{2}\arcsin(x)\right).$

$f(x) = \sqrt{1-x^2} \Rightarrow F(x) = \frac{1}{2}x\cdot\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) + c.$

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$c\cdot g(x)$	$c\cdot g'(x)$
$g(ax+b)$	$a\cdot g'(ax+b)$
$g(x)+h(x)$	$g'(x)+h'(x)$
$g(x)\cdot h(x)$	$g'(x)\cdot h(x)+g(x)\cdot h'(x)$
$\frac{t(x)}{n(x)}$	$\frac{n(x)\cdot t'(x)+n'(x)\cdot t(x)}{(n(x))^2}$
e^x	e^x
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)}=1+\tan^2(x)$
$\arctan(x)$	$\frac{1}{x^2+1}$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$
$\ln\|x\|$	$\frac{1}{x}$

α	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\sin(\alpha)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\alpha)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\tan(\alpha)$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	↯

$f(x)$	$F(x)$
x^n	$\frac{1}{n+1}x^{n+1}$
e^x	e^x
$\sin(x)$	$-\cos(x)+c$
$\cos(x)$	$\sin(x)+c$
$1+\tan^2(x)$ of $\frac{1}{\cos^2(x)}$	$\tan(x)+c$
$g(ax+b)$	$\frac{1}{a}\cdot G(ax+b)+c$
$\frac{1}{x^2+1}$	$\arctan(x)+c$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(x)+c$
$\ln(x)$	$x\ln(x)-x$

$g(x)\mathrm{d}f(x) = \mathrm{d}\left(g(x)\cdot f(x)\right) - f(x)\mathrm{d}g(x)$

booglengte op $[a,b] = \int_a^b \sqrt{1+\left(f'(x)\right)^2}\,\mathrm{d}x$